A New System for Rounding Numbers

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The conventional rules for rounding numbers after calculation are inadequate. They do not consider the significance of the digits dropped, and they lead to inequitable treatment of numbers rounded up relative to numbers truncated (rounded down).

Effect of Significance of the Dropped Digits

According to the conventional rule, final calculated values should retain all certain digits and the first uncertain digit. This is reasonable: All digits less certain than the first uncertain digit are rejected. They are regarded as so uncertain that there is a good chance they are incorrect. Not only can they misrepresent precision, they can do it with digits that are wrong. Both certainty and wrongness are related to the digits in the specific, calculated value. Since the digits cannot guarantee the accuracy of the data on which their calculation was based, they cannot guarantee the accuracy of the calculated value.

Consider the division of 5.0 ± 0.1 by 3.0 ± 0.1, in which each number has two significant digits. The digit in the units place is certain, and the digit in the tenths place is uncertain by 1. Our calculator gives only one number, 1.666666, as the quotient, but there are really nine possibilities.

5.1 / 3.1 = 1.64516
5.1 / 3.0 = 1.70000
5.1 / 2.9 = 1.75862 (maximum)
5.0 / 3.1 = 1.61290
5.0 / 3.0 = 1.66666 (best estimate)
5.0 / 2.9 = 1.72413
4.9 / 3.1 = 1.58064 (minimum)
4.9 / 3.0 = 1.63333
4.9 / 2.9 = 1.68965

Although the digit in the tenths place is 5–7, the digit in the hundredths place is 0–8, which includes almost all digits in the decimal system. We know the digit in the tenths place is ±1, but we do not know the digit in the hundredths place at all. It should not be retained, and we properly drop it.

Now we do a strange thing. Having ascertained that the digit in the hundredths place is so uncertain that we should eliminate it, we use it as a criterion to decide how large the retained number should be. Thus, on the basis of this very shaky digit, we change the maximum, the best estimate, and the minimum values to 1.8, 1.7, and 1.6 (i.e., 1.7 ± 0.1).

This is not logical. If the digit in the hundredths place is so uncertain that it should be eliminated, why should it serve as a basis for deciding the size of the number? The number should be truncated; it should never be rounded up based on the magnitude of a nonsignificant digit. The numbers should remain 1.7, 1.6, and 1.5 (i.e., 1.6 ± 0.1).

Effect of Magnitude of the Dropped Digits

Should we drop digits that are significant? If so, when should we round up and when should we truncate?

The procedure for rounding is again strange. Generally, a number such as $1.2x$ is thought to have nine possible values: 1.21–1.29. The first four numbers, 1.21–1.24, are the bottom half of the range, and the top four numbers, 1.26–1.29, are the top half. Thus, we truncate if the dropped digit is 1–4, and raise if that digit is 6–9. This creates a problem when the digit to be dropped is 5, so we make rules. "When the digit being dropped is 5, truncate if the previous digit is even, but raise if it is odd." Alternatively, "Truncate if the previous digit is odd, but raise if it is even."

What really happens with these rules? For the number $1.xy$, we truncate to $1.x$ if y is less than 5, regardless of the value of x. For any of the five numbers $1.x0$–$1.x4$, we choose $1.x$. For any of the four numbers $1.x6$–$1.x9$, we choose $1.(x + 1)$. When y is 5, though, we round up half the time and truncate half the time. Overall, we truncate 55% of the time and round up 45% of the time. This inequity arises because the conventional rules do not cover 1.20. Obviously, $1.2x$ has ten possible values (1.20–1.29), not nine (1.21–1.29).

There is a better solution. When y is 0–4, truncate; when 5–9, raise. Since, both rounding up and truncation occur five times, this is more equitable. Also, we do not have to worry about even or odd digits, or about which of the previous rules to follow.

Try a sample calculation to compare the rules.

With the conventional rule:
For 1.15–1.25, choose 1.2, and for 1.26–1.34, choose 1.3.
This gives 1.2 eleven times and 1.3 nine times.
Alternatively, 1.16–1.24 become 1.2, and 1.25–1.35 become 1.3.
This still yields nine one way and eleven the other.

By the system I propose:
For 1.15–1.24, choose 1.2; for 1.25–1.34, choose 1.3.
This yields both 1.2 and 1.3 ten times.

Conclusion

The present rules for rounding calculated numbers are inadequate, illogical, and inequitable—even overly complex. Both the significance and the size of the dropped digits should be considered when rounding. I propose a new and better rule: "If the first (leftmost) digit dropped is significant and is 5–9, round up. Otherwise, truncate."