Major Concepts

- Ideal gas
  - Equations of State
- Real Gases
  - Virial Series
  - Vdw
  - Critical Behavior
- Kinetic Model of Gases
  - Velocity and speed distribution
  - Types of average speed
  - Collision of gases and pressure

You should review

- Definition of a mole
- SI Units
- Ideal gas law
- Critical point definition

Force	$F = ma = m\left(\dfrac{dv}{dt}\right) = m\left(\dfrac{d^2 r}{dt^2}\right) = \dfrac{dp}{dt}$
Momentum	$p = mv = m\dfrac{dr}{dt}$
Pressure	$P = F / A = \left(\dfrac{dp}{dt}\right) / A$

Mathematical tools

- Power series https://en.wikipedia.org/wiki/Power_series
- MacLaurin series
  http://mathworld.wolfram.com/MaclaurinSeries.html
- Inflection point
  https://www.khanacademy.org/math/differential-calculus/analyzing-func-with-calc-dc/points-of-inflection-dc/v/inflection-points
- Simple derivatives
  https://www.mathsisfun.com/calculus/derivatives-rules.html
- Solving cubic equations
  https://en.wikipedia.org/wiki/Cubic_function
- Spherical Polar coordinates
- Center of Mass Coordinates
- Using Tables of Integrals

Variables of state: specify the state of a system

n, amount of substance   P, pressure

V, volume it occupies   T, temperature

$P = F/A$

$$T/\text{K} = \theta/°\text{C} + 273.15$$

Table 1A.1 Pressure units*

Name	Symbol	Value
pascal	1 Pa	$1\ \text{N m}^{-2}$, $1\ \text{kg m}^{-1}\ \text{s}^{-2}$
bar	1 bar	10^5 Pa
atmosphere	1 atm	101.325 kPa
torr	1 Torr	(101 325/760) Pa = 133.32… Pa
millimetres of mercury	1 mmHg	133.322… Pa
pounds per square inch	1 psi	6.894 757… kPa

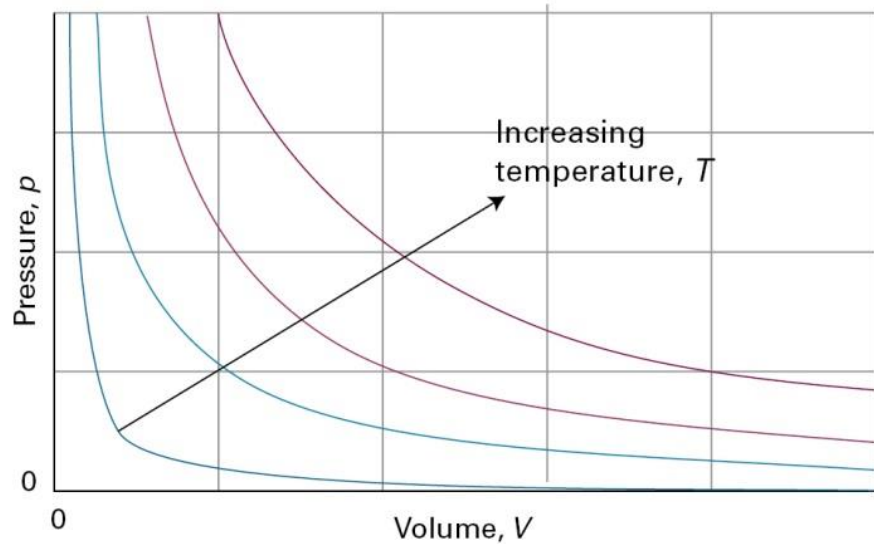* Values in bold are exact.

Equation of state

$$P = f(n, V, T)$$

Figure 1A.2 The pressure–volume dependence of a fixed amount of perfect gas at different temperatures. Each curve is a hyperbola (pV = constant) and is called an isotherm.

Empirical Laws:

Boyle's Law:	PV = constant	constant n, T
Charles Law:	V = constant x T	constant n, P
	P = constant x T	constant n, V
Avogadro's Principle:	V = constant x n	constant P, T

Limiting Laws– strictly true only at P→0

Ideal Gas Law (Perfect Gas Law)   *PV = nRT*

Table 1A.2 The gas constant ($R = N_A k$)

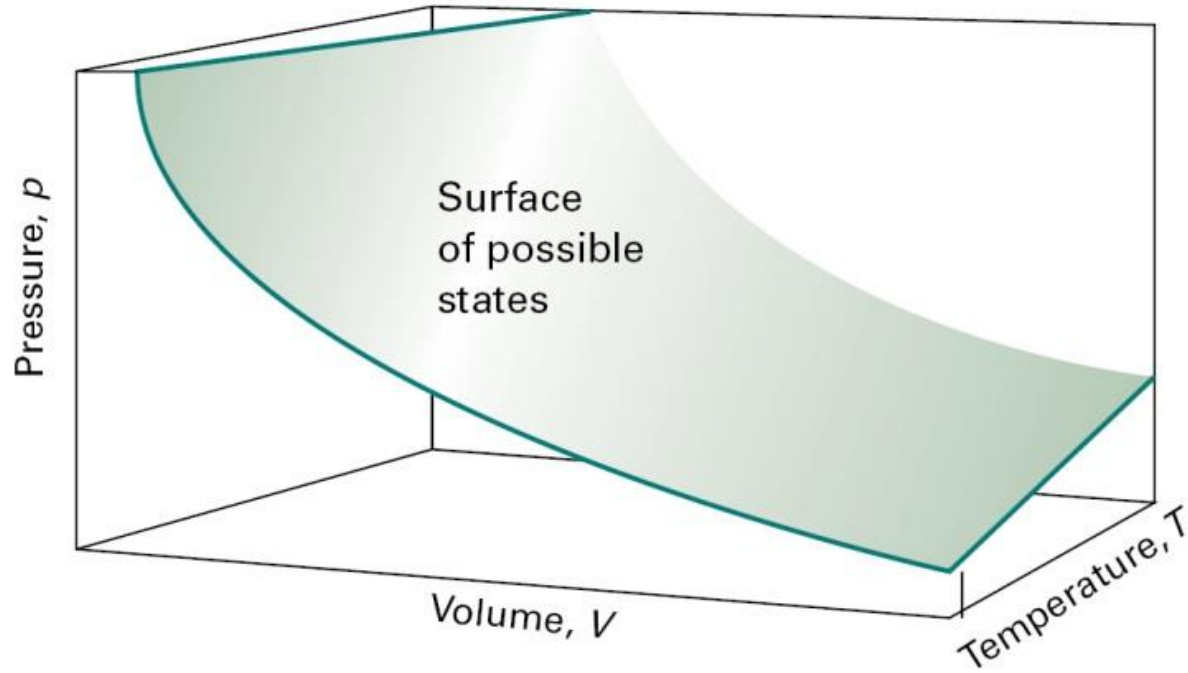| R	
8.314 47	J K^{-1} mol^{-1}
8.205 74×10^{-2}	dm^3 atm K^{-1} mol^{-1}
8.314 47×10^{-2}	dm^3 bar K^{-1} mol^{-1}
8.314 47	Pa m^3 K^{-1} mol^{-1}
62.364	dm^3 Torr K^{-1} mol^{-1}
1.987 21	cal K^{-1} mol^{-1}

Figure 1A.6 A region of the *p,V,T* surface of a fixed amount of perfect gas. The points forming the surface represent the only states of the gas that can exist.
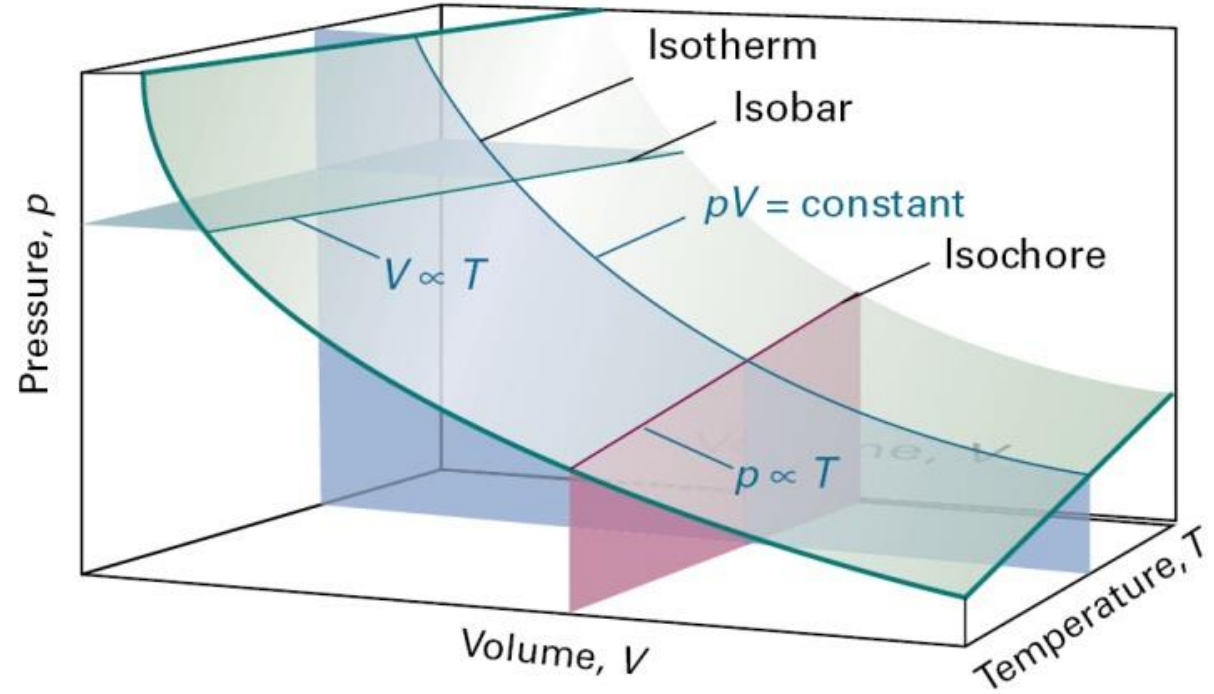
Figure 1A.7 Sections through the surface shown in Fig. 1A.6 at constant temperature give the isotherms shown in Fig. 1A.2, the isobars shown in Fig. 1A.4, and the isochores shown in Fig. 1A.5.
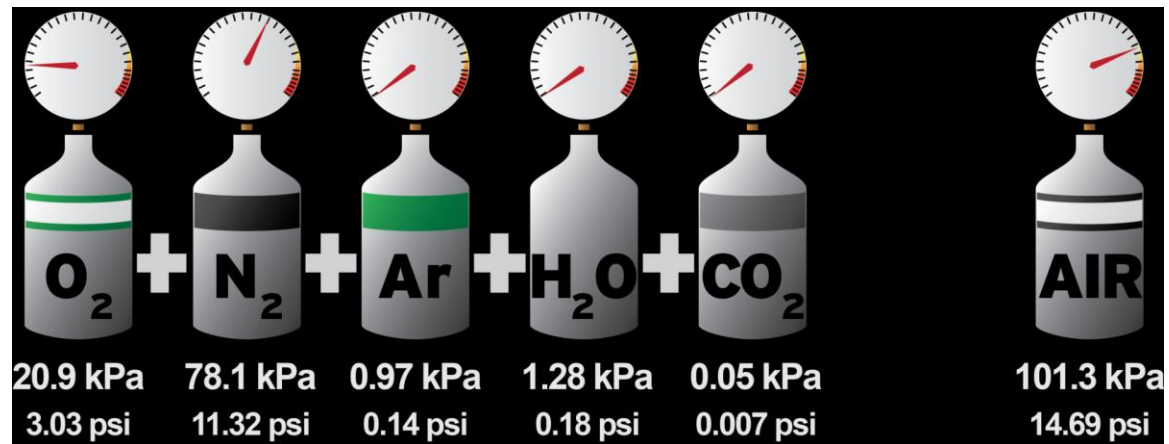
Partial pressure of mixture of gases, Dalton's Law

- Total pressure of gas mixture = sum of partial pressures of the gases
- Partial pressure of the gas is proportion to the **mole fraction** (x_i)

$$P_i = x_i \cdot P$$

$$P = \Sigma\, P_i, \quad P_i V = n_i RT$$

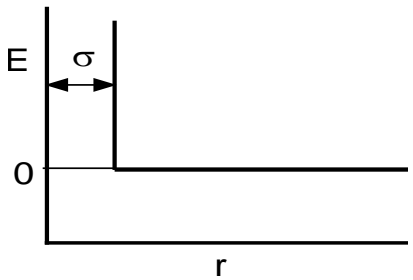P_i: partial press of gas i, n_i: amount of gas i in moles

O_2	N_2	Ar	H_2O	CO_2	AIR
20.9 kPa	78.1 kPa	0.97 kPa	1.28 kPa	0.05 kPa	101.3 kPa
3.03 psi	11.32 psi	0.14 psi	0.18 psi	0.007 psi	14.69 psi

By Max Dodge - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=35993393

- Real gases have molecular interactions, both attractive and repulsive.
- Short range – strongly repulsive
- Long range – weakly attractive

Pairwise potential function $U(r)$ gives potential energy between a pair of molecules as a function of distance between them, r.
We'll ignore orientation dependence (bad for polar molecules; ok for spherical)

$U = 0 \qquad r > \sigma$
$U = \infty \qquad r \leq \sigma$

$U = 0 \qquad R\sigma < r < \infty$
$U = -\varepsilon \qquad \sigma < r < R\sigma$
$U = \infty \qquad 0 < r < \sigma$

$U = -\varepsilon\left(\frac{\sigma}{r}\right)^{n} \quad \sigma < r < \infty$
$U = \infty \qquad 0 < r < \sigma$

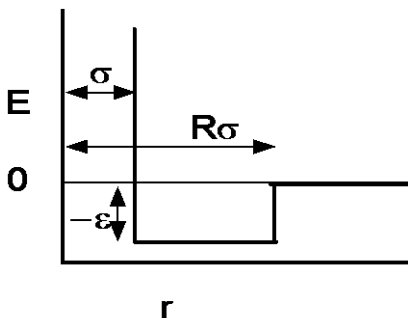$\phi_{LJ}(r)$

Repulsive $+A/r^{12}$

Attractive $-B/r^{6}$
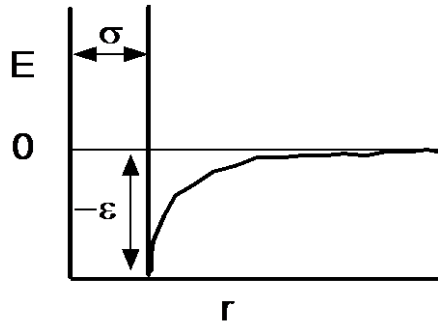
Hard Sphere
Simplest model
Parameters
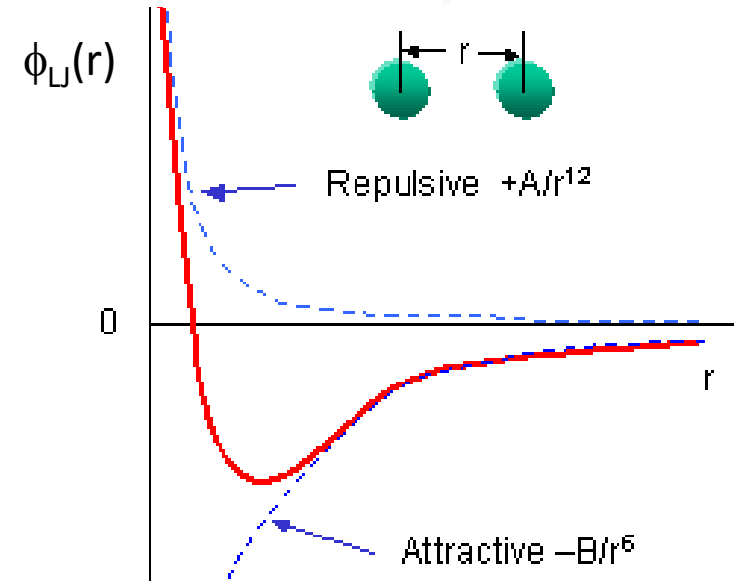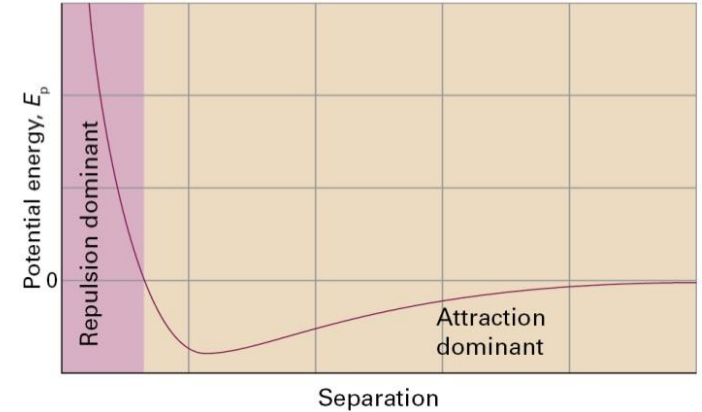- Well depth ε
- diameter σ (hard sphere)

Square Well
has a potential well

Sutherland
Better represents long-term attraction

$$U(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

Lennard-Jones Potential

At critical point, phase boundary between liquid and gas disappears and satisfies:

$$\left(\frac{\partial p}{\partial V}\right)_T = \left(\frac{\partial^2 p}{\partial V^2}\right)_T = 0$$

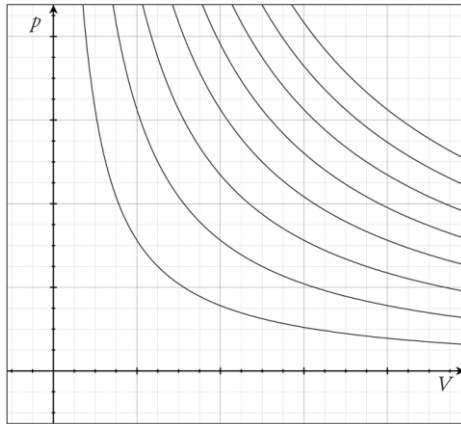ideal gas isotherms do not predict critical behavior

Table 1C.2* Critical constants of gases

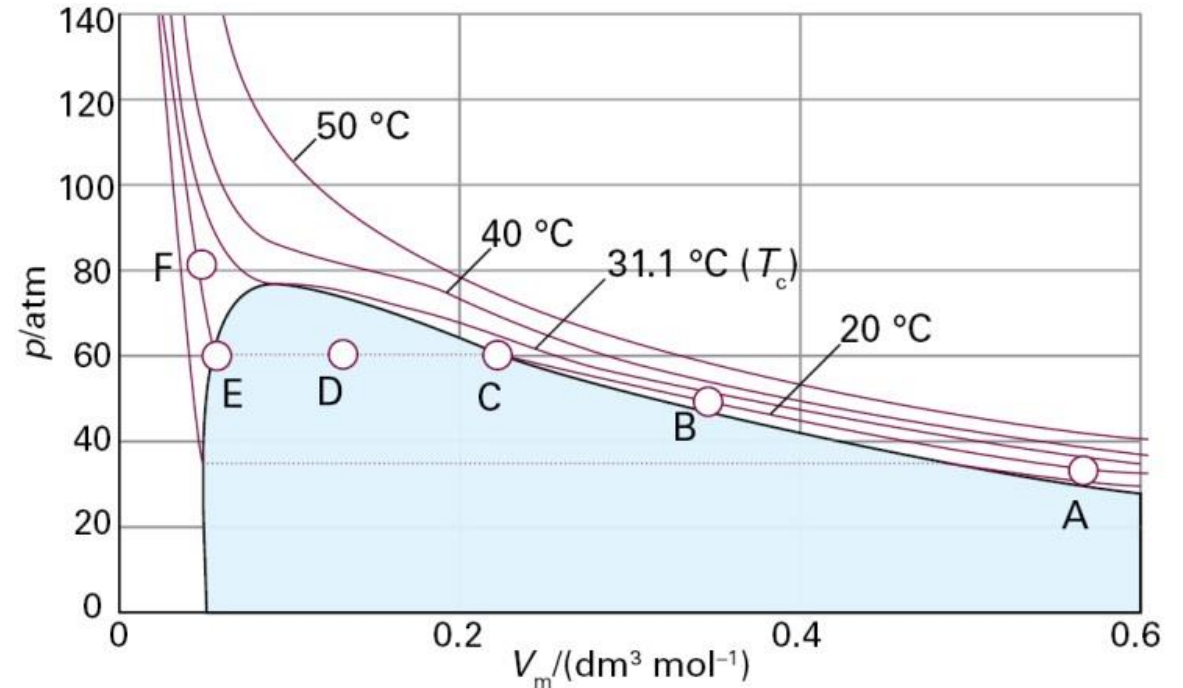|	p_c/atm	V_c/(cm³ mol⁻¹)	T_c/K	Z_c	T_B/K
Ar	48.0	75.3	150.7	0.292	411.5
CO₂	72.9	94.0	304.2	0.274	714.8
He	2.26	57.8	5.2	0.305	22.64
O₂	50.14	78.0	154.8	0.308	405.9

* More values are given in the *Resource section*.

Figure 1C.2 Experimental isotherms of carbon dioxide at several temperatures. The 'critical isotherm', the isotherm at the critical temperature, is at 31.1 °C.

Need to modify ideal gas equation of state to accommodate non-ideality of the real gas.

$$z \equiv \frac{PV_m}{RT}$$

Compressibility
For ideal gas, z = 1
Deviation from 1 is measure of non-ideality
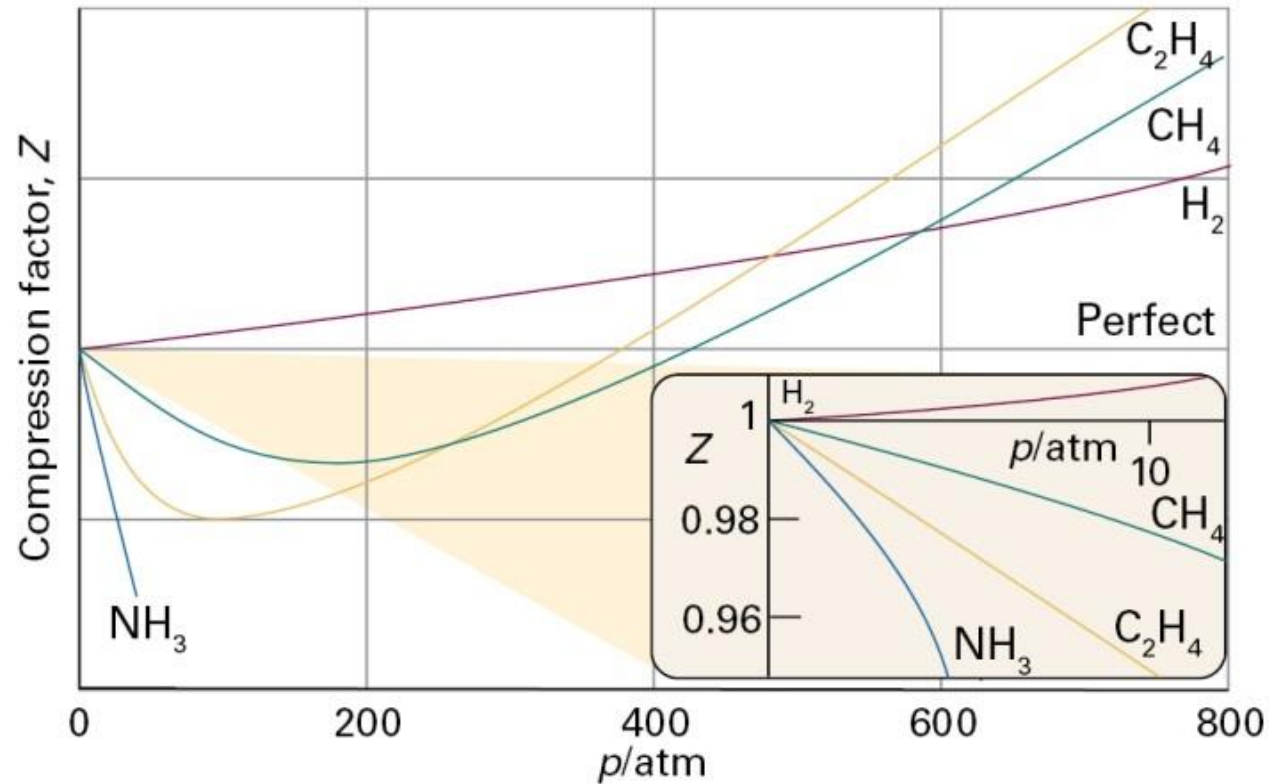

Figure 1C.3 The variation of the compression factor, Z, with pressure for several gases at 0 °C. A perfect gas has $Z=1$ at all pressures. Notice that, although the curves approach 1 as $p \to 0$, they do so with different slopes.

Virial equation of state: one way to discusses real gases with moderate deviation from ideality

$$PV_m = RT(1 + B'P + C'P^2 \dots)$$

$$PV_m = RT\left(1 + \frac{B}{V_m} + \frac{C}{V_m^2} + \cdots\right)$$

Coefficients used to calculate intermolecular interactions
1st term (1) represents non-interacting particles
2nd term (B) represents *pairs* of molecules
3rd term (C) represents *trios* of molecules (usually less important than B)

Table 1C.1* Second virial coefficients, $B/(cm^3\,mol^{-1})$

	Temperature	
	273 K	600 K
Ar	−21.7	11.9
CO_2	−149.7	−12.4
N_2	−10.5	21.7
Xe	−153.7	−19.6

* More values are given in the *Resource section*.

------------- Math Sidebar -----------
Any analytic function can be fit by an infinite power series

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots$$

Only useful if series can be truncated after a few terms i.e. $a_{n+}1 \ll a_n$ or $x \ll 1$

Boyle Temperature, T_B
B(T) can be +, - or 0
T when B(T)=0 is called Boyle temperature.
Gas behaves ideally at this temp.

van der Waals equation of state

$$P = \frac{RT}{V_m - b} - \frac{a}{V_m^2}$$

or

$$\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$$

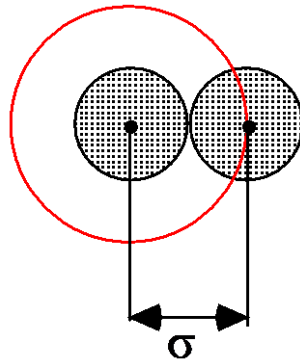Accounts for intermolecular attraction

Accounts for volume of molecule

a, b constants depend on gas.

a ($Pa.m^6.mol^{-2}$) -- accounts for attractive force which effectively reduce pressure.

b ($m^3.mole^{-1}$) -- accounts for the fact that molecules occupy space (repulsive forces)

Volume available = container vol. - volume of molecules

excluded volume:

Table 1C.3* van der Waals coefficients

	$a/(\text{atm dm}^6\,\text{mol}^{-2})$	$b/(10^{-2}\,\text{dm}^3\,\text{mol}^{-1})$
Ar	1.337	3.20
CO_2	3.610	4.29
He	0.0341	2.38
Xe	4.137	5.16

* More values are given in the *Resource section*.

$$b = \frac{2\pi L\sigma^3}{3}$$

~~2~~σ -- diameter of spheres of excluded volume

σ -- diameter of molecules

L -- number of molecules (L/2 pairs)

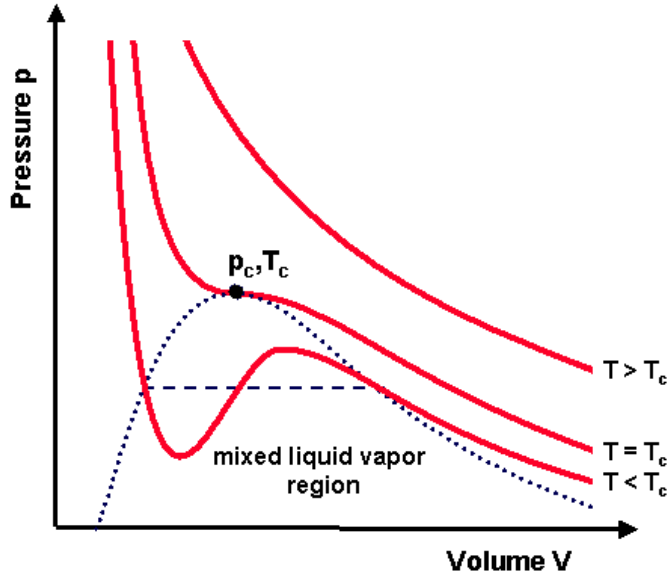Excluded volume per pair of atoms: $\qquad V = \frac{4\pi\sigma^3}{3}$

vdw gas isotherms qualitatively predict critical behavior. P_c, V_c, T_c can be expressed in terms of vdw constants

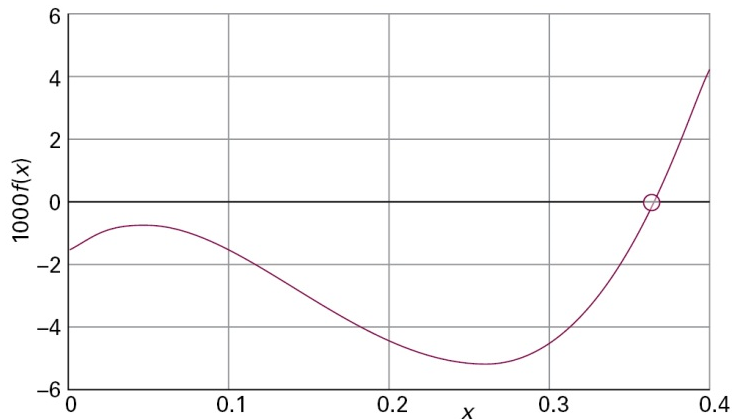(c) C. Rose-Petruck, Brown University, 7-Jan-99, Chem 201 #1

At tie-line, P = Vapor Pressure
Liq. & Gas in Equilibrium

V_c=3b $P_c = \dfrac{a}{27b^2}$ $T_c = \dfrac{27}{Rb}$

vdw is cubic in volume, so has 3 roots
2 of the roots account for volumes of two phases
1 -- large volume (gas)
2 -- small volume (liquid)
3-- not a stable root (dP/dV is positive)
better for *qualitative* information than *quantitative*
e.g. NH_3 at 350 K   vdw predicts vp of 61 atm; actual is 38.4 atm

Figure 1C.6 The graphical solution of the cubic equation for *V* in *Example* 1C.1.

$$V_m{}^3 - \left(b + \frac{RT}{P}\right)V_m{}^2 + \left(\frac{a}{P}\right)V_m - \left(\frac{ab}{P}\right) = 0$$

- *Law of Corresponding States* -- properties of different gases are similar at their respective critical points.
- Scaling properties to critical parameters produces universal behavior.

Reduced Constants

$$T_r = \frac{T}{T_c} \qquad P_r = \frac{P}{P_c} \qquad V_r = \frac{V}{V_c}$$
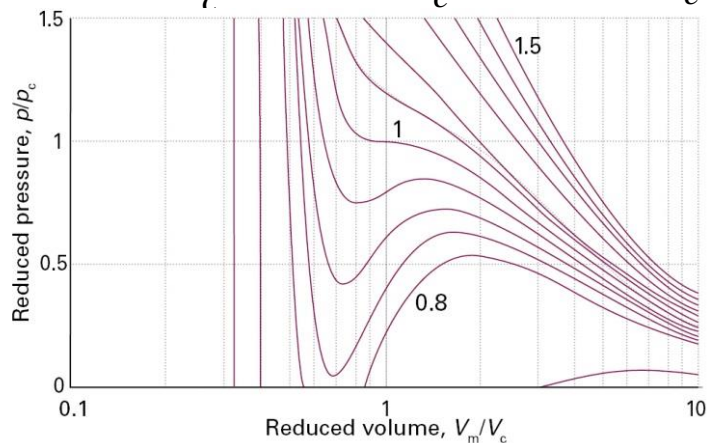
Figure 1C.8 van der Waals isotherms at several values of T/T_c. Compare these curves with those in Fig. 1C.2. The van der Waals loops are normally replaced by horizontal straight lines. The critical isotherm is the isotherm for $T/T_c = 1$.

Figure 1C.9 The compression factors of four of the gases shown in Fig. 1C.3 plotted using reduced variables. The curves are labelled with the reduced temperature $T_r = T/T_c$. The use of reduced variables organizes the data on to single curves.

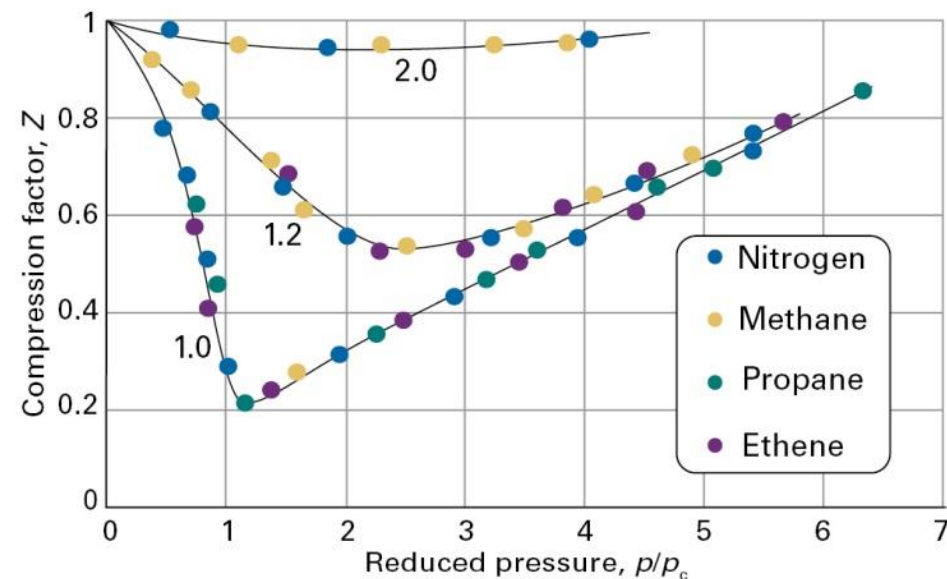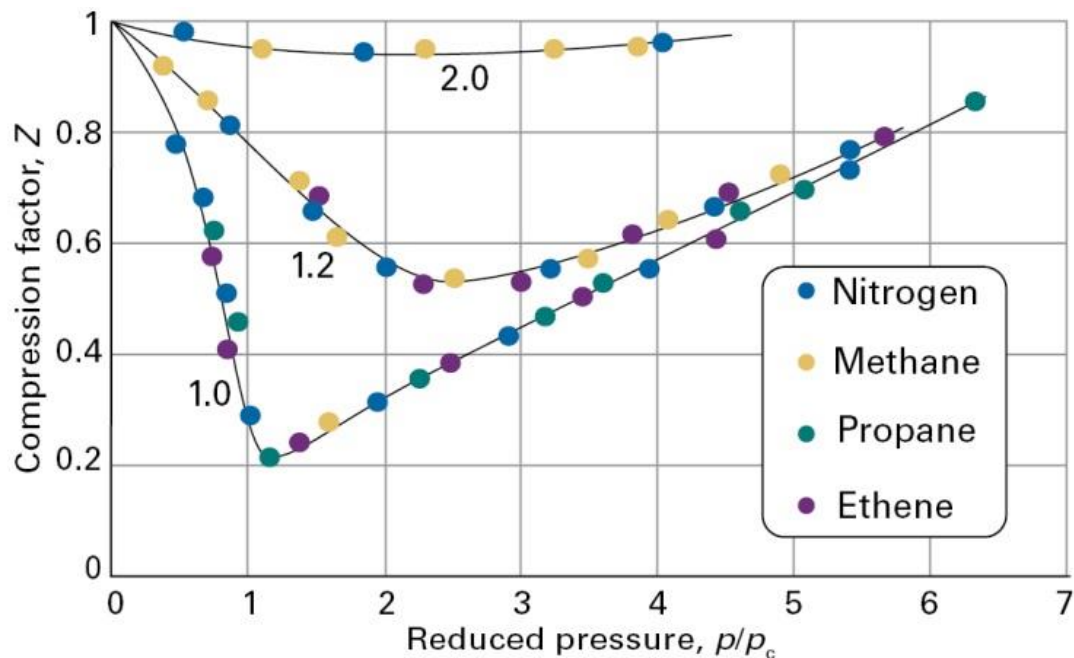Figure 1C.9 The compression factors of four of the gases shown in Fig. 1C.3 plotted using reduced variables. The curves are labelled with the reduced temperature $T_r = T/T_c$. The use of reduced variables organizes the data on to single curves.

Table 1C.2* Critical constants of gases

	p_c/atm	$V_c/(cm^3\ mol^{-1})$	T_c/K	Z_c	T_B/K
Ar	48.0	75.3	150.7	0.292	411.5
CO_2	72.9	94.0	304.2	0.274	714.8
He	2.26	57.8	5.2	0.305	22.64
O_2	50.14	78.0	154.8	0.308	405.9

* More values are given in the *Resource section*.

Table 1C.4 Selected equations of state

|	Equation	Reduced form*	Critical constants p_c	V_c	T_c
Perfect gas	$p = \dfrac{nRT}{V}$				
van der Waals	$p = \dfrac{nRT}{V-nb} - \dfrac{n^2 a}{V^2}$	$p_r = \dfrac{8T_r}{3V_r - 1} - \dfrac{3}{V_r^2}$	$\dfrac{a}{27b^2}$	$3b$	$\dfrac{8a}{27bR}$
Berthelot	$p = \dfrac{nRT}{V-nb} - \dfrac{n^2 a}{TV^2}$	$p_r = \dfrac{8T_r}{3V_r - 1} - \dfrac{3}{T_r V_r^2}$	$\dfrac{1}{12}\left(\dfrac{2aR}{3b^3}\right)^{1/2}$	$3b$	$\dfrac{2}{3}\left(\dfrac{2a}{3bR}\right)^{1/2}$
Dieterici	$p = \dfrac{nRTe^{-aRTV/n}}{V-nb}$	$p_r = \dfrac{T_r e^{2(1-1/T_r V_r)}}{2V_r - 1}$	$\dfrac{a}{4e^2 b^2}$	$2b$	$\dfrac{a}{4bR}$
Virial	$p = \dfrac{nRT}{V}\left\{1 + \dfrac{nB(T)}{V} + \dfrac{n^2 C(T)}{V^2} + \cdots\right\}$				

* Reduced variables are defined in Section 1C.2(c). Equations of state are sometimes expressed in terms of the molar volume, $V_m = V/n$.

Vdw and other gas law constants can be related to Virial Coefficients B,C, and D

e.g. vdw: $B = b - \dfrac{a}{RT}$ $C = b^2$ $D = b^3$

Kinetic Theory of Gases

Kinetic Theory of Gases - assumes that the only contribution to the energy of gas is from kinetic energy of molecules (i.e. - potential energy of interactions is negligible)
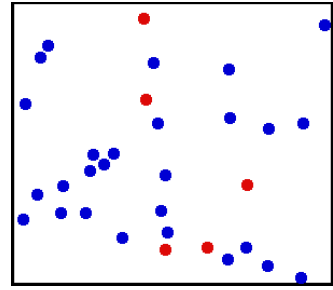
Based on 3 assumptions:

1. **Molecules have mass, m, and diameter, d**

Characterized by velocity*, v, and momentum (p = mv) or kinetic energy (KE = 1/2mv²)

Important quantity: Number density: n* = N/V   [Using ideal gas law n*=PN$_A$/RT (n=N/N$_A$)]

2. **Size of molecules is negligible** (diameters much smaller that average distance travelled between collisions)

3. **Molecules do not interact other than perfectly elastic collisions** when separation of their centers is equal to d.

Will use kinetic theory to find energy and speed of molecules.
===

velocity - vector quantity; has direction (θ, φ) and magnitude(|v|) $v_x = \dfrac{dx}{dt}$ $v_y = \dfrac{dy}{dt}$ $v_z = \dfrac{dz}{dt}$

   can be positive or negative

speed -- always positive $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ $v_x = v\sin\theta\cos\phi$ $v_y = v\sin\theta\sin\phi$ $v_z = v\cos\theta$
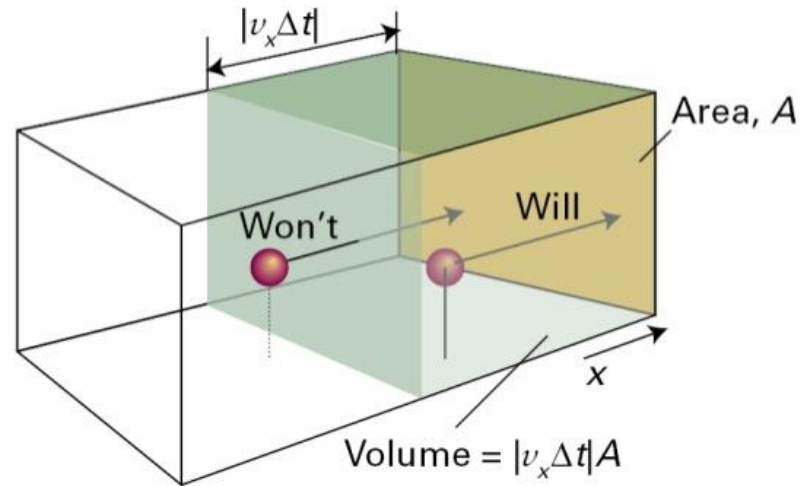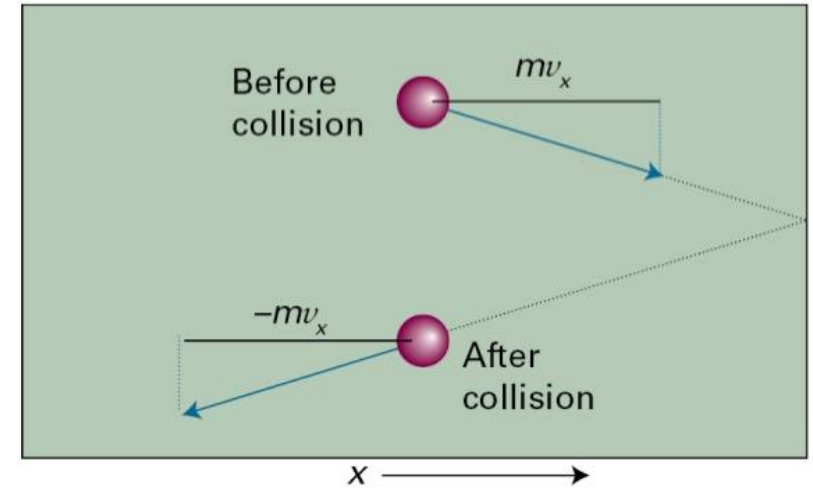
Consider that you have a wall in the yz plane.
Molecules with a velocity in x direction will collide with wall.
A molecule with v_x will collide w/ wall w/in time Δt.

Will molecule collide with wall?
"Construct" a box with height $= |v_x| \Delta t$ and cross-sectional area $=A$
Volume $= A|v_x|\Delta t$

• So *# collisions* **with velocity** v_x = n* **A**$|v_x|$**Δt** f(v_x) (Remember, n* is molecules/unit volume) where f(v_x) is the fraction of molecules having v_x

• ***Total*** *collisions* = n*A ΔtΣ$|v_x|$f(v_x)   summed over positive velocities; must know f(v_x) to find
But, can find P and speed of gas without f(v_x)

Consider momentum: Molecule heading toward wall

•*momentum **before** collision*: $p_x = mv_x$ $p_y = mv_y$ $p_z = mv_z$
        after *collision*: $p_x = -m|v_x|$ $p_y = mv_y$ $p_z = m|v_z|$

•***Difference*** *in momenta for each collision*: $\Delta p = 2m|v_x|$

•*Momenta of all collisions **of** v_x* = Δp/collision•#collisions = 2m$|v_x|$n*AΔt$|v_x|$f(v_x) = 2mn*AΔt$|v_x|^2$f(v_x)

•*Momenta for **all** collisions* = 2mn*A ΔtΣ$|v_x|^2$f(v_x) summed over positive velocities

$\sum_{positive} v_x^2 f(v_x) = \frac{1}{2} \sum_{all} v_x^2 f(v_x)$ b/c on average, as many molecules have positive as negative velocity

- Define *average squared velocity* $\langle v_x^2 \rangle = \sum_{all} v_x^2 f(v_x)$ (angle brackets used to denote an average)

$$\sum_{positive} v_x^2 f(v_x) = \frac{1}{2} \langle v_x^2 \rangle$$

- *Change in momenta for all collisions* $2\text{mn*A } \Delta t \Sigma |v_x|^2 f(v_x) : \Delta p = mn^* A \Delta t \langle v_x^2 \rangle$
- Pressure of a Gas   Pressure = Force/unit Area

        Force (F) = mass (m) • acceleration (a) = m • dv/dt = dp/dt ~Δp/Δt
        Therefore, P ~ Δp/ΔtA $P = mn^* \langle v_x^2 \rangle$

- *Pressure in terms of velocity* $PV = mnN_A \langle v_x^2 \rangle = nM \langle v_x^2 \rangle$

Use ideal gas $nRT = PV = nM \langle v_x^2 \rangle$. So, $\langle v_x^2 \rangle = \frac{RT}{M}$

 relates a microscopic quantity (velocity) to macroscopic quantity (temperature)
Choose other walls would get same result for v_y and v_z $v^2 = v_x^2 + v_y^2 + v_z^2$

 Average: $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle = \frac{3RT}{M}$ • *root-mean-square speed*

$$\boxed{u = \sqrt{\frac{3RT}{M}}}$$ $\langle v^2 \rangle^{1/2} \equiv u$

Recap & Reminders

- **Virial Series and virial coefficients used to describe real gases**

$$PV_m = RT \left(1 + \frac{B}{V_m} + \frac{C}{V_m^2} + \cdots \right)$$

- ***Van der Waals equation of state corrects for non-idealities with constants a and b***

$$P = \frac{RT}{V_m - b} - \frac{a}{V_m^2}$$

- ***Reduced parameters (Tr, Vr, Pr) can be used to compare different gases since many properties scale as a ratio of their critical points. Principle of Corresponding States***

$$T_r = \frac{T}{T_c} \qquad P_r = \frac{P}{P_c} \qquad V_r = \frac{V}{V_c}$$

- ***Other equations of states are used. These models of gas laws can be related to each other.***

- ***Kinetic theory of gases assumes gases have mass m, diameter, d and velocity v.***

$$KE = \tfrac{1}{2}mv^2 \qquad Average \quad KE = \frac{m\langle v^2 \rangle}{2} = \frac{m}{2}\frac{(3RT)}{M} \qquad Average \quad KE = \frac{3}{2}k_bT$$

m - mass per atom

M - mass per mole

Boltzmann's constant $\quad k_b = R / L$

M = mL

Only accounts for translational energy

Now, to determine number of collisions, need to know f(v_x) -- the probability distribution function

Start by considering an average $\langle x \rangle = \dfrac{\sum n_i x_i}{\sum n_i}$

<x> --average property -- will use this notation often!

n_i-number of molecules having property $n_T = \sum n_i$ -- total number of molecules

Probability of finding a certain property is $f_i = \dfrac{n_i}{n_T}$

average: $\langle x \rangle = \sum f_i x_i$ mean square: $\langle x^2 \rangle = \sum f_i x^2_i$

$\langle x^2 \rangle^{\frac{1}{2}}$ **(root-mean-square)** is different from $\langle x \rangle$ **(average).**
     Mean square is related to degree which numbers differ from average.
• If x is continuous: convert sum to   integral
     $dn = n_T f(x)dx$ dn -- number of items having property between x and x+dx
               n_T -- total number of particles
               f(x) probability distribution function

$f(x)dx = \dfrac{dn}{n_T}$

Example: $f(x) = \dfrac{1}{2\sqrt{\pi}} \exp\left[\dfrac{-(x-21)^2}{4}\right]$

1) The sum of all fractions is 1. Such a function is said to be *normalized.*

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

2) Average value of x is given by $\langle x \rangle = \int_{-\infty}^{\infty} xf(x)dx$

Compare to $\langle x \rangle = \sum f_i x_i$

3) Mean Square of x is given by $\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 f(x)dx$

Compare to $\langle x^2 \rangle = \sum f_i x^2{}_i$

4) To find the probability that a particle has a property between x_1 and x_2

$$P(x_1, x_2) = \int_{x_1}^{x_2} f(x)dx$$

Velocity distribution function

$$f(v_x)dv_x = \sqrt{\frac{m}{2\pi k_b T}} \exp\left(-\frac{mv_x}{2k_b T}\right) dv_x$$

Speed distribution function
   Justification 1B.2

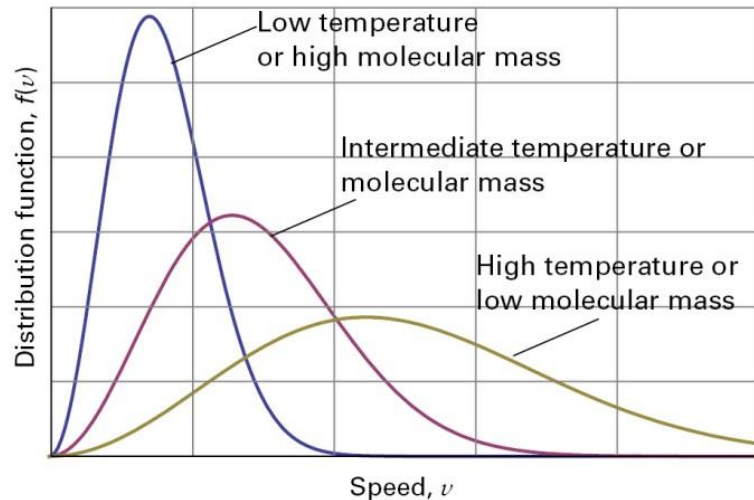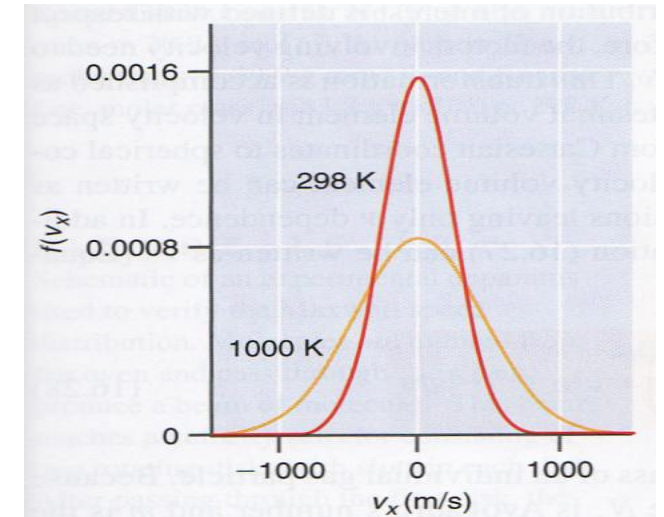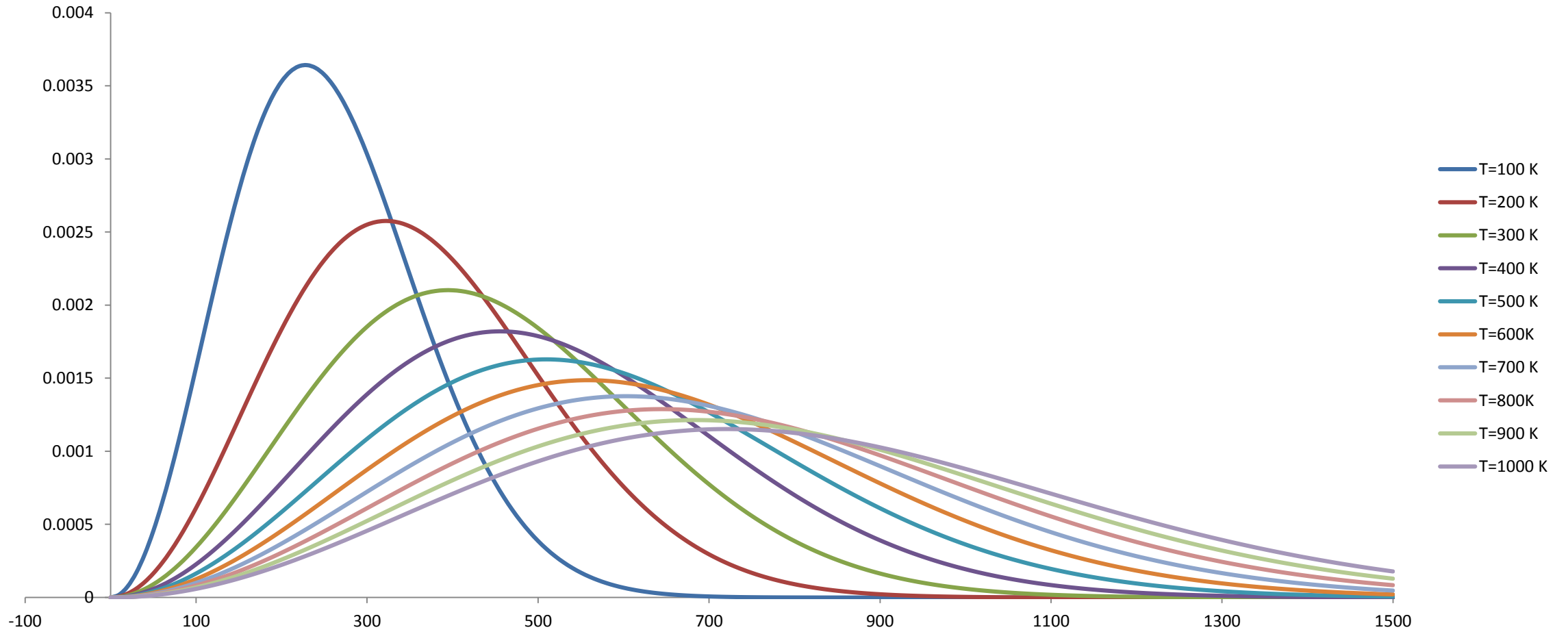$$F(v)dv = 4\pi\left(\frac{m}{2\pi k_b T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_b T}\right)v^2 dv$$

- Decaying exponential
  - fraction of molecules with high speeds is small
- Mass factor in exponential
  - heavy molecules unlikely to be found at high speeds
- Inverse T in exponential
  - greater fraction of molecules at high speeds at high T
- Factor of v^2
  - Fraction of molecules with low speeds will be small

Figure 1B.4 The distribution of molecular speeds with temperature and molar mass. Note that the most probable speed (corresponding to the peak of the distribution) increases with temperature and with decreasing molar mass, and simultaneously the distribution becomes broader.

Distribution of speeds for O₂

Probable and Average Speeds

$$F(v)dv = 4\pi \left(\frac{m}{2\pi k_b T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_b T}\right)v^2 dv$$

$$f(v) = 4\pi \left[\frac{M}{2\pi RT}\right]^{3/2} v^2 \exp\left[\frac{-Mv^2}{2RT}\right]$$

↓ m or ↑ T, maxima occur at higher v.

- Most Probable Speed -- take derivative of F(v)dv and set equal to 0.(Do this!) $\boxed{v_p = \sqrt{\frac{2k_b T}{m}}}$

- Average speed $\boxed{\bar{v} = \langle v \rangle = \sqrt{\frac{8RT}{\pi M}}}$

From $\langle v \rangle = \int_0^\infty vF(v)dv$ -- starts at 0, because there are no negative speeds (Do this!)

$$v_p = \sqrt{\frac{2RT}{M}}$$

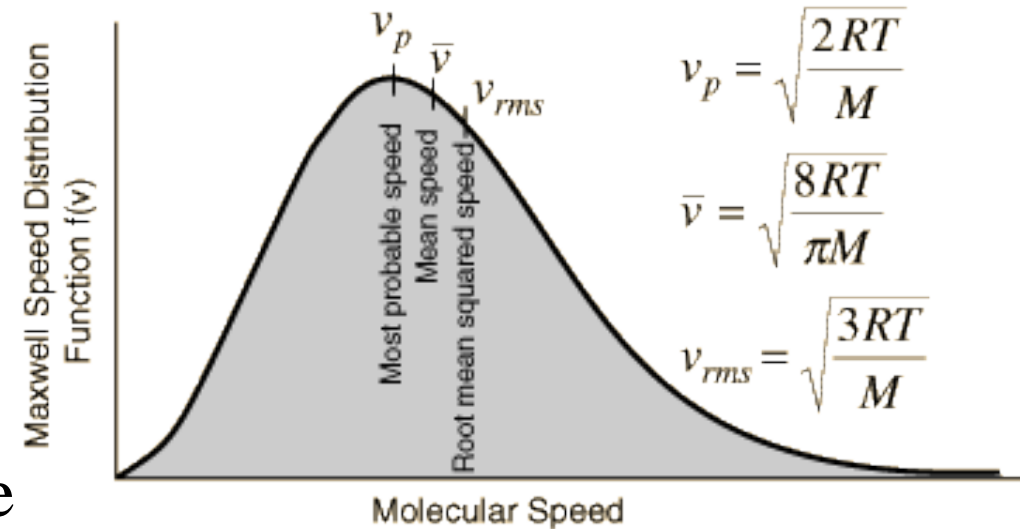$$\bar{v} = \sqrt{\frac{8RT}{\pi M}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

- Fraction of molecules having speeds between v_1 and v_2

$$P(v_1, v_2) = \int_{v_1}^{v_2} F(v)dv = 4\pi \left(\frac{m}{2\pi k_b T}\right)^{3/2} \int_{v_1}^{v_2} \exp\left(-\frac{mv^2}{2k_b T}\right)v^2 dv$$

Must evaluate numerically—can use unitless equation with $w \equiv v/\sqrt{2k_b T/m}$ (m in kg)

$$P(v_1, v_2) = \frac{4}{\sqrt{\pi}} \int_{w_1}^{w_2} \exp\left(-w^2\right)w^2 dw$$

Want to determine the **collision frequency** and **total number of collisions** between gas molecules.

Assume molecules are hard spheres with *diameters* of d.

Two molecules will collide if they approach w/in distance d.

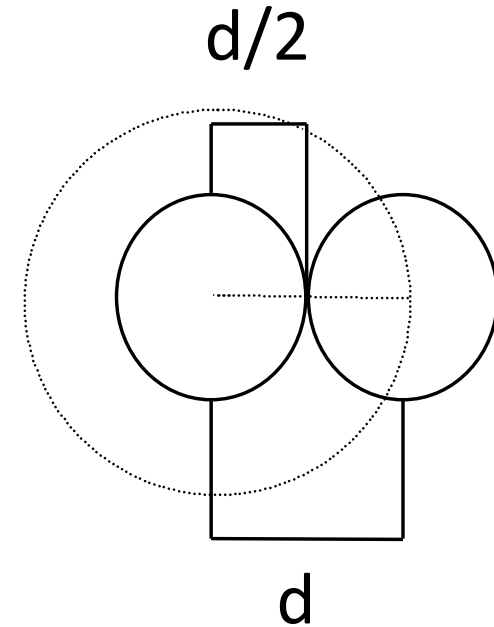A molecule moving through space at speed v will sweep out cylinder of volume V in Δt.

Any molecule whose center is in this volume will be hit.

Miss

$v_{rel}\Delta t$

Hit

d

d

Area, σ
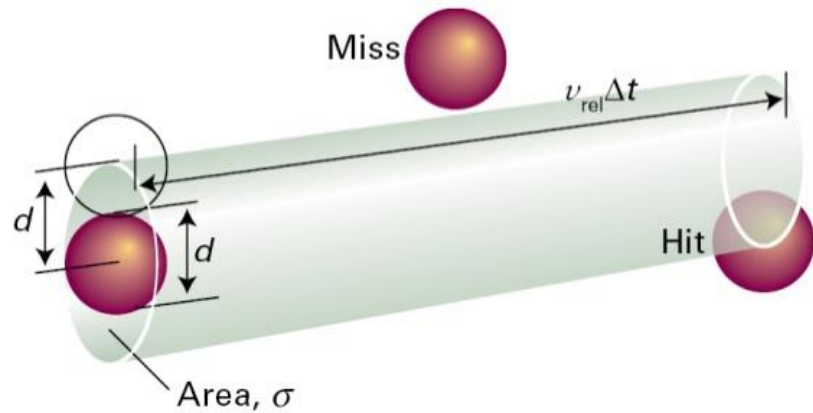
$$V = \pi d^2 v\Delta t$$

Area : $\sigma = \pi d^2$

Length: $v\Delta t$ (m/s)s=m

d/2

d

collision cross section: $\sigma = \pi d^2$

cross-sectional area of an imaginary sphere surrounding molecule into which the center of another molecule cannot penetrate

Table 1B.1* Collision cross-sections, σ/nm^2

	σ/nm^2
C_6H_6	0.88
CO_2	0.52
He	0.21
N_2	0.43

* More values are given in the *Resource section*.

Collision frequency, Z, depends on molecule's collisional cross-section, speed, & density of targets. $Z = \sigma v n^*$

density of targets in cylinder depends on **number density** (molecules / unit volume), n*
Using ideal gas (and n = N/N$_A$), $n^* = \dfrac{N}{V} = \dfrac{PN_A}{RT}$

Not all molecules moving at same v. $\qquad\qquad\qquad \bar{v} = \int\limits_0^\infty vF(v)dv = \sqrt{\dfrac{8RT}{\pi M}}$
The average speed:

Really want **average relative velocity** b/c molecule is hitting a moving target (another molecule) not a fixed target (e.g. wall) (See Figure 1B.8)

$$v_{rel} = \left(\dfrac{8kT}{\pi\mu}\right)^{1/2} \qquad \mu = \dfrac{m_A m_B}{m_{A+}m_B} \qquad v_{rel} = \sqrt{2}\,\bar{v} \qquad \text{For identical molecules}$$

$$Z = \sigma v_{rel}n^* \qquad\qquad z = \dfrac{\sigma v_{rel}P}{kT}$$

Mean free path (λ) is the average distance traveled by a molecule between collisions

$$\lambda = \frac{v_{rel}}{z}$$

Using z, $\lambda = \dfrac{k_b T}{\sigma P}$

Get σ from Table 1B.1 or resource section

Table 1B.1* Collision cross-sections, σ/nm^2

	σ/nm^2
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Compare λ to σ: Gas is mostly empty space!

Ideal Gas, PV = nRT
- Compressibility (z), Virial Series, and Coefficients
- How used; Boyle Temperature

Gas Mixtures

Critical Behavior and Law of Corresponding States

Modeling of Intermolecular Interactions in Real Gases

Real Gas Laws: vdw, others

Relationship between intermolecular forces, gas constants, critical constants, and virial coefficients

Kinetic Molecular Theory

Molecular Speeds

Maxwell-Boltzmann distribution

Collision Frequency

Mean Free Path