Problem Set 1b
Due Thursday, September 17, 2009

Problems from Cotton:

Chapter 2: 2.1, 2.3, 2.6. Problem 2.4 is incorrect; there are groups for which $A^2 = B^2 \neq E$ for two of the symmetry operations $A$ and $B$. Find two such counterexamples from point groups.

Chapter 3: A3.2, A3.3, B3.2, B3.4, B3.10
Set C: 3, 6, 10, 13, 14, 15
Set D: 2 (this is a lousy drawing of the ferrocene core), 3, 4, 11, 12
Set E: 1, 5 (neither staggered nor eclipsed), 8, 12

Additional Problems:

(1) Assign the point group symmetry to the following objects and molecules in the conformations indicated. For the molecules, assume that the conformation of the N-H, C-H, etc. bonds do not destroy symmetry elements otherwise present.


b) The surface shown in two views below.

c) The helical segment shown below.

d) TEMPO (2,2,6,6-tetramethylpiperidin e-1-oxyl):
$C_9H_{18}NO_8[\overset{2-}{\text{C\vphantom{1}N}}]$.

e) $B_2H_6$

f) $[\text{Mo(CN)}_7]^{4-}$
$1^{st}$ isomer

g) $[\text{Mo(CN)}_7]^{4-}$
$2^{nd}$ isomer
(2) As shown in Cotton (p. 72), the matrix $C_\varphi = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ represents a clockwise rotation of angle $\varphi$ about the $z$-axis (viewed down the axis) in the basis

\[
\hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \; \hat{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \; \hat{z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.
\]

a) By means of multiplication of these vectors by $C_\varphi$, verify this fact. (Draw pictures that show how the rotation works for the case with $\varphi = 3\pi/4$, or 135°.)

b) Use the method given in part (a) to find the inverse of $C_\varphi$ and verify your answer by means of matrix multiplication.

c) Use the method given in the handout to find the eigenvalue(s) and eigenvectors of $C_\varphi$. Explain the geometrical significance of your results.

(3) (a) List all the symmetry operations of a four-bladed propeller and identify the point group for which you have listed the operations.

(b) Work out a multiplication table for this symmetry group (check your result using the rearrangement theorem).

(c) If the propeller blades are flattened so that their surfaces don’t “tilt” (that is, they look the same from the front and back), what “supergroup” would apply?
(d) For the supergroup you gave in part (c), work out a multiplication table (check your result using the rearrangement theorem), divide the group elements into classes. If in doubt, check your results using the definition of conjugate elements and the multiplication table.

(e) What subgroup do you obtain if you include only the proper rotation elements for the group of an equilateral triangle?

(4) Consider all the proper symmetry operations of a regular pentagon. (Proper operations are just the identity and the rotations; they are operations that one can actually perform with a physical object.)

(a) Identify the group and write a list of all the operations.

(b) Work out the multiplication table of the group (use the rearrangement theorem to check your result.)

(c) From the geometric nature of the operations, divide the group into classes. For one nontrivial class (i.e., not the identity), verify that all the possible similarity transforms you can write generate a member of that class.

(d) Write down all the subgroups of the complete group. Identify all the invariant subgroups. (An invariant subgroup is a one that consists entirely of complete classes of the original group.)

(5) Look through Cotton’s character tables and identify all the point groups in which every symmetry operation commutes with every other symmetry operation.