Boyle's Sound relationship between $P$ & $V$ (Fig 12.4)

- pressure is \textit{inversely proportional} to volume (or vice versa)
- as one goes up, other goes down

Boyle's law $P_1V_1 = P_2V_2$ (at constant $n, T$)
(analogy - decrease room size)

ex. at 25°C, He occupies 250 mL @ pressure 760 torr.
what volume would it occupy at 2 atm (same temp)?

\[
\begin{align*}
760 \text{ torr} \times \frac{1 \text{ atm}}{760 \text{ torr}} &= 1 \text{ atm} \\
(250 \text{ mL})(1 \text{ atm}) &= (2 \text{ atm})V_2 \\
V_2 &= 125 \text{ mL}
\end{align*}
\]

We kept temperature constant. What if we changed it?

(Jacques) Charles studied the relationship between volume & temperature ($V$ & $T$)
(balloonist)

- increase temperature & watch for changes in volume (Fig 12.5)
  - as temp increases, volume increases
  - works for several different pressures, not just ordinary atmospheric pressure.

relative to a constant, we write

\[
\frac{V_1}{T_1} = \text{constant} \quad \text{temperature is directly proportional to volume (or vice versa)}
\]

Charles Law \[
\frac{V_1}{T_1} = \frac{V_2}{T_2}
\]
(demonstration, coke can)

- Fig 12.9, Fig 12.12, portocle view
potential example

\[ \text{H}_2 \text{ occupies } 100 \text{ mL @ } 25^\circ \text{C & 740 torr.} \]

What volume does it occupy @ 50\(^\circ\)C (same pressure)?

\[ 273 + 25 = 298 \text{ K} \quad \quad 273 + 50 = 323 \text{ K} \]

\[ \frac{(100 \text{ mL})}{298 \text{ K}} = \frac{V_2}{323 \text{ K}} \quad \quad V_2 = 108 \text{ mL} \]

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P related to V

T related to V

can tie all three together and use the combined gas law

\[ \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \text{(less memorization)} \]

(looking at previous example, constant pressure cancels to form Charles' law)

(or you can plug it in on both sides)

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Reference conditions (for convenience)

**Standard temperature & Pressure (STP)** - 0\(^\circ\)C (273.15 K)

1 atm (760 torr)

**ex.** a sample of CO\(_2\) occupies 350 mL @ 45\(^\circ\)C, pressure = 910 torr.

What volume would it occupy at STP?

\[ \frac{(910 \text{ torr})(350 \text{ mL})}{(273 + 45 \text{ K})} = \frac{(760 \text{ torr})(V_2)}{(273 \text{ K})} \]

\[ V_2 \approx 360 \text{ mL} \]
in all our calculations so far, we have assumed that the amount of gas is constant. what is effect if we increase/decrease quantity? (analogy - people in room)

as # of people increases, pressure increases - add more people to standance volume increases - add more seats

must be directly proportional to P & V (like temp)

Avogadro's Law (experimental basis)

\[ \frac{V_1}{n_1} = \frac{V_2}{n_2} \]

standard molar volume of ideal gas = 22.414 L/mole.

(at STP)

(ex-see next page)

combining with other laws

\[ \frac{PV}{nT} = \text{constant} \]

\[ R = \text{value depends on units} \]

\[ = 0.0821 \text{ atm}. \frac{\text{L}}{\text{mol}. \text{K}} \]

\[ = 62.4 \text{ bar}. \frac{\text{L}}{\text{mol}. \text{K}} \]

rearrange: \[ PV = nRT \] ideal gas law (mentioned previously)

why ideal? "real" gases do not behave exactly as this law dictates (fuzzy shapes, interactions with walls & each other) but it's a very good 1st approximation
ex. What volume does 45.0g of N₂ occupy at STP?

What is density of N₂ at STP?

\[ \text{mmN}_2 = 28.1 \text{g/mol} \]

\[ 45.0 \times \frac{\text{mol}}{28.1\text{g}} = 1.60 \text{mol N}_2 \]

\[ \frac{V_i}{n_i} = \frac{V_f}{n_f} \]

\[ \frac{22.414 \text{L}}{1\text{mol}} = \frac{V_f}{1.60\text{mol}} \]

\[ V_f = 35.9 \text{L} \]

\[ \text{density} = \frac{45.0\text{g}}{35.9 \times 10^3 \text{mL}} = 1.25 \times 10^{-3} \frac{\text{g}}{\text{mL}} \]

or \[ = 1.25 \text{g/L} \]

ex. What volume does 48g of methane occupy at 140°C under a pressure of 1280 torr?

\[ \text{mw CH}_4 = 16\text{g/mol} \]

\[ 48g \times \frac{1\text{mol}}{16\text{g}} = 3.0 \text{ mol CH}_4 \]

\[ 140°C + 273 = 413K \]

\[ \frac{PN}{nRT} = (1280 \text{ torr})(V) = (3.0)(62.4)(413K) \]

\[ V = 60.4 \text{ L} \]

ex. What is the density of a sample of CO₂ at 150°C & 740 torr?

\[ \text{density} = \frac{\text{g}}{\text{L}} \]

\[ PV = nRT \]

\[ \frac{740}{(62.4)(423)} = \frac{n}{V} \]

\[ \frac{740}{(62.4)(423)} = \frac{740}{(62.4)(423)} \times \frac{44.0\text{g/mol}}{1 \text{ mol}} = 1.23 \frac{\text{g}}{\text{L}} \]
we can rearrange \( P \cdot V = nRT \)
\[ \frac{m \cdot (P)}{R \cdot T} = \frac{n}{\gamma} \]
\[ \frac{(mw)(P)}{RT} \text{ Density for a general equation} \]

(Wendt, notes, 12-11, for exam example)

**Dalton's Law of Partial Pressure**

In a mixture of gases, each gas exerts the pressure that it would if it occupied the volume alone.

\[ P_{\text{total}} = P_A + P_B + P_C \]
\[ \text{partial pressure} \]

**Example:** 100 mL of \( H_2 \), measured at 25°C & 2.0 atm pressure, & 100 mL of \( O_2 \), " " & 1.0 atm pressure, were combined into one container of volume 100 mL. What would be the total pressure in the container?

\[ P_{\text{total}} = P_{H_2} + P_{O_2} \]
\[ = 2.0 \text{ atm} + 1.0 \text{ atm} \]
\[ = 3.0 \text{ atm} \] (note, \( V \) & \( T \) were constant)

\[ \begin{bmatrix} n_{\text{total}} = n_A + n_B + n_C \\ PV = n_{\text{total}} RT \end{bmatrix} \text{ alternate method.} \]

**Example:** Calculate pressure exerted by mixture of 6.00g \( H_2 \), 12.0g \( He \), & 24.0g \( N_2 \) in 20.0L at 23°C.

\[ n_{\text{total}} = \frac{6.00 \text{ g}}{2.0 \text{ g}} + \frac{12.0 \text{ g}}{4.0 \text{ g}} + \frac{24.0 \text{ g}}{28.0 \text{ g}} = 2.97 \text{ mol } H_2 + 3.00 \text{ mol } He + 0.957 \text{ mol } N_2 \]
\[ N_{\text{total}} = 6.83 \text{ mol gas} \]

\[ P_{\text{total}} = \frac{\text{NRT}}{V} = \frac{6.83 \times (0.0821)(23 + 273)}{20.04} \approx 8.30 \text{ atm} \]

\[ P_B = P_{\text{total}} - P_A \quad \text{ (for a system of 2 gases A & B)} \]
Quit 7 key

0.382 g of an unknown gas is contained in a 200 mL flask. The temperature is 100°C and the pressure is 736 torr.

What is the molar mass of the gas?

\[ PV = nRT \]

We know \( P, V, R, \text{ and } T \).

We can solve for \( n \) (moles)

Molar mass is g/mole.

We know \( g \) and we solve for \( n \) mole.

\[
\frac{n}{RT} = \frac{(736 \text{ torr})(0.200 \text{ L})}{(62.4 \text{ torr} \cdot \text{L} \cdot \text{mol}^{-1} \cdot \text{K}^{-1})(273 + 100 \text{ K})}
\]

\( R \) has L units, must convert to liters!

\( R \) has K units, must convert to kelvin!

\[ n = 6.32 \times 10^{-3} \text{ mol} \]

Molar mass = \[
\frac{0.382 \text{ g}}{6.32 \times 10^{-3} \text{ mol}} = 60.4 \text{ g/mol}
\]

\[ \frac{\text{g}}{\text{mol}} = \frac{\text{g}}{	ext{mm}^3 \text{molar mass}} \]

\[ \text{all in one step: } 1 \text{ mole} = \frac{\text{g}}{(\text{g/mole})} = \frac{\text{g}}{\text{mm}} \]

\[ PV = nRT = PV = \left( \frac{\text{g}}{\text{mm}} \right) RT \]

\[ \text{mm} = \frac{\text{g}RT}{PV} \]

Use this equation