Molecular Vibrations

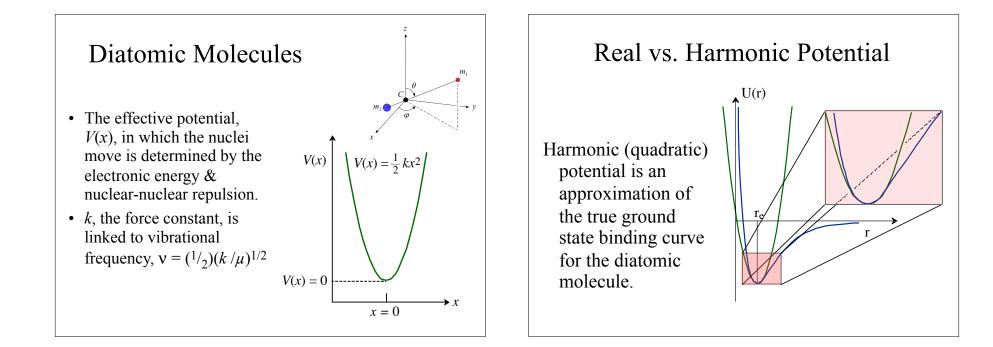
Read:

- Cotton, *Chemical Applications...*, Chapter 10.
- Harris & Bertolucci, *Symmetry and Spectroscopy...*, Chapter 3, pp. 93-117; 135-167; 170-201.

Diatomic Molecules

The harmonic oscillator Hamiltonian is

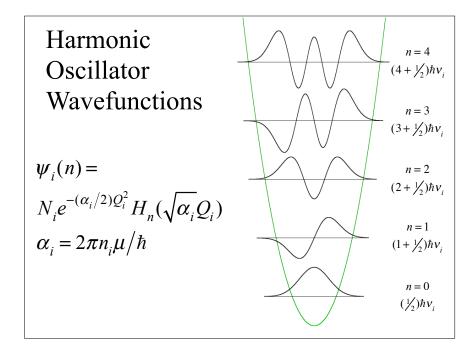
 $\left(-\frac{h^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\varphi = E\varphi \qquad V(x) = \frac{1}{2}kx^2$ $x = \text{ distance from } x_0$ $\left(\text{Hooke's Law: Force} = F = -\frac{dV(x)}{dx} = -kx\right)$



Normal Coordinate for diatomics

The Hamiltonian is rewritten (change of variables to mass weighted coordinate):

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 \implies -\hbar^2\frac{d^2}{dq^2} + \frac{1}{2}\lambda q^2$$
$$\mu = \frac{m_1m_2}{m_1 + m_2} \qquad q = \sqrt{\mu}x ; \quad \lambda = k/\mu \quad ; \quad V(q) = \frac{1}{2}\lambda q^2$$
frequency $v = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{1}{2\pi}\sqrt{\lambda}$



Normal Coordinates

• Expressed in terms of normal coordinates, the vibrational potential energy function is diagonal (contains no cross terms):

$$U = U_{eq} + \frac{1}{2} \sum_{k=1}^{3N-6} \lambda_k Q_k^2$$

There are no terms involving $Q_i Q_j$

Normal Coordinates for Polyatomics

• For polyatomics, there exists a set of normal coordinates in which the <u>vibrational Hamiltonian</u> is separable

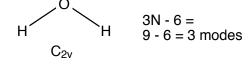
$$\mathcal{H}_{vib} = \sum_{k=1}^{3N-6} \mathcal{H}_k; \quad \mathcal{H}_k = -\frac{\hbar^2}{2} \frac{\partial^2}{\partial Q_k^2} + \frac{1}{2} \lambda_k Q_k^2$$

 Q_k are mass-weighted coordinates

- <u>Energies are summed</u>: each mode: $E_i = (n_i + \frac{1}{2})hv_i$ Total: $E_{tot}^{vib} = \sum_{i=1}^{3N-6} E_i$
- <u>Wavefunctions are products</u> of wavefunctions of modes: $\psi_1(n_1)\psi_2(n_2)\cdots\psi_{3N-6}(n_{3N-6}) = \prod_{i=1}^{3N-6}\psi_i(n_i)$

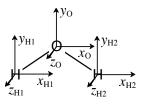
Normal Modes

The "normal modes" of vibration are those which correspond to motions along "normal coordinates". Such normal coordinates transform as irreducible representations of the applicable point group, but are generally combinations of symmetry adapted internal coordinates. For example, consider H_2O . To what I.R.s do the normal modes belong?

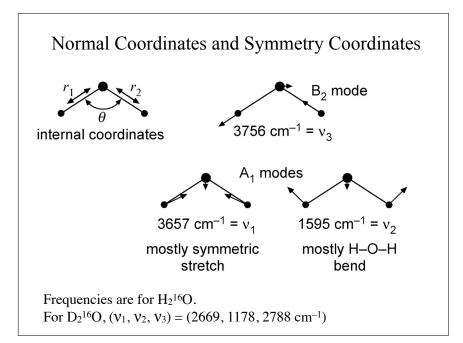


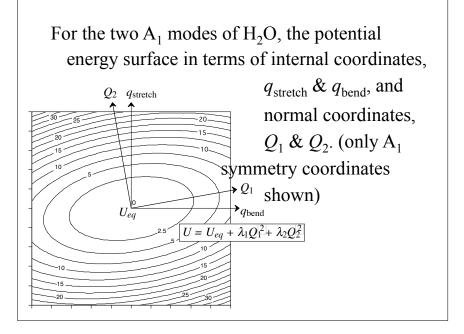
Determining Normal Mode Symmetries

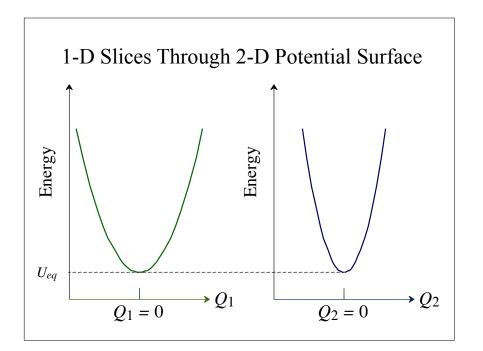
• Find reducible rep. for all nuclear motions



 Eliminate translations and rotations (6 for nonlinear molecules, 5 for linear molecules). This guarantees that <u>normal modes involve</u> <u>no net translation or rotation of center of</u> <u>mass</u>.







Harmonic Oscillator Wavefunctions

• For each mode, i, ψ_i has the form

$$\psi_{i}(n) = N_{i}e^{-(\alpha_{i}/2)Q_{i}^{2}}H_{n}(\sqrt{\alpha_{i}}Q_{i}); \ \alpha_{i} = 2\pi n_{i}\mu/\hbar$$

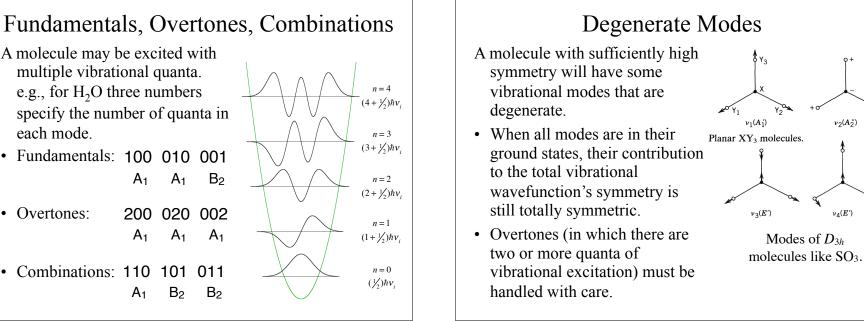
$$H_{0}(x) = 1; \ H_{1}(x) = 2x; \ H_{2}(x) = 4x^{2} - 2;$$

$$H_{3}(x) = 8x^{3} - 12x; \ H_{4}(x) = 16x^{4} - 48x^{2} + 12$$
For even(odd) $n, H_{n}(x)$ is an even(odd) function.
$$H_{n}(\sqrt{\alpha_{n}}Q_{n}) = \hat{R} \rightarrow H_{n}(\sqrt{\alpha_{n}}Q_{n}) \text{ for even } n$$

 $H_n(\sqrt{\alpha_i Q_i}) \xrightarrow{n} H_n(\sqrt{\alpha_i Q_i})$ for even *n*. and $H_n(\sqrt{\alpha_i}Q_i)$ transforms like Q_i for odd n.

v2(A2)

 $\nu_4(E')$



A molecule may be excited with

multiple vibrational quanta. e.g., for H₂O three numbers specify the number of quanta in each mode. • Fundamentals: 100 010 001 A₁ A₁ • Overtones: 200 020 002 A₁ A₁ • Combinations: 110 101 011 A₁ B_2

If two modes, *i* & *j*, form a basis for a degenerate representation, then λ_i and λ_j are equal (i.e., the modes have the same frequency). Then,

$$\hat{R}Q_i = aQ_i + bQ_j ; \hat{R}Q_j = cQ_i + dQ_j$$

$$\Gamma(R) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is an orthogonal matrix}$$

$$\therefore a^2 + b^2 = c^2 + d^2 = a^2 + c^2 = c^2 + d^2 = 1;$$

$$ac + bd = ab + cd = 0.$$

$$\lambda_i (\hat{R}Q_i)^2 + \lambda_i (\hat{R}Q_j)^2 = \lambda_i \begin{bmatrix} a^2Q_i^2 + b^2Q_j^2 + 2abQ_iQ_j \\ + c^2Q_i^2 + d^2Q_j^2 + 2cdQ_iQ_j \end{bmatrix}$$

$$= \lambda_i (Q_i^2 + Q_j^2) \qquad (ab + cd)Q_iQ_j = 0$$

The Result:

The total potential, U, is not changed under the action of a symmetry operation. Total wavefunction is unchanged too:

when $\alpha_i = \alpha_j$,

$$e^{-\frac{\alpha_{i}}{2}Q_{i}^{2}}e^{-\frac{\alpha_{i}}{2}Q_{j}^{2}} = e^{-\frac{\alpha_{i}}{2}(Q_{i}^{2}+Q_{j}^{2})} \xrightarrow{\hat{R}} e^{-\frac{\alpha_{i}}{2}(Q_{i}^{2}+Q_{j}^{2})}$$

 $\psi_{\rm vib.}$ is therefore unchanged in the ground state (At the bottom p. 325 in Cotton, this is not at all clear.)

- <u>Conclusion</u>: All molecules' vibrational wavefunctions belong to the totally symmetric representation for the appropriate point group when then molecule is in its vibrational ground state (no excited modes).
- <u>Fundamentals</u>: One vibrational quantum is created

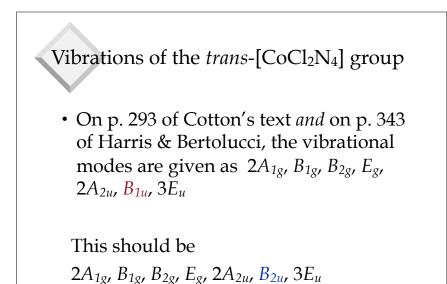
$$\prod_{i=1}^{3N-6} \psi_i(0) \to \left\{ \prod_{i\neq j}^{3N-6} \psi_i(0) \right\} \psi_j(1)$$

— symmetry of vibrational wavefunction determined by the j^{th} mode.

 CoL_6 modes in $[trans-CoCl_2(en)_2]^+$

- To understand the electronic spectra of transition metal complexes, it turns out that the vibrational modes have to be considered.
- Electronic spectrum of [*trans*-CoCl₂(en)₄]⁺
 is shown in Fig. 9.13 in Cotton
- Determine the vibrational modes of the [trans-CoCl₂N₄] grouping.

D_{4h}	E		$C_2(C_4^2)$						$2\sigma_v$			2 2 2
A_{lg}	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
4_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	R _z	
B_{1g}	1	-1	1	1	-1	1	-1	1	1	-1		$x^2 - y^2$
B_{2g}	1	-1	1	-1	1	1	-1	1	-1	1		xy
B_{2g} E_{g}	2	0	-2	0	0	2	0	-2	0	0	(R_x, R_y)	(xz, yz)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		
4_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	z	
B_{1u}	1	-1	1	1	-1	-1	1	-1	-1	1		
B_{2u}	1	-1	1	-1	1	-1	1	-1	1	-1		
B_{2u} E_u	2	0	-2	0	0	-2	0	2	0	0	(x, y)	

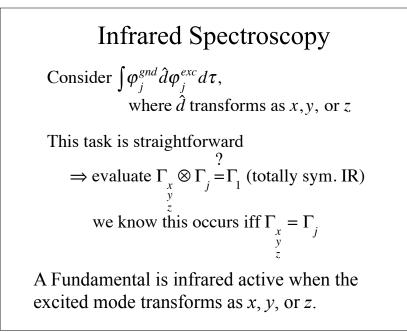


Example: CO stretching modes for Cr(CO)₆ (Mn(CO)₆⁺, V(CO)₆⁻)

O_h	Ε	8 <i>C</i> ₃	$3C_2 = (= C_4^2)$	$6C_4$	$6C_2$	i	8 <i>S</i> ₆	$3\sigma_h$	$6S_4$	$6\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1		
Ē	2	-1	2	0	0	2	-1	2	0	0		$(2z^2 - x^2 - y^2, x^2 - y^2)$
$egin{array}{c} A_{2g} & & \\ E_g & & \\ T_{1g} & & \end{array}$	3	0	-1	1	-1	3	0	$^{-1}$	1	$^{-1}$	(R_x, R_y, R_z)	
	3	0	-1	-1	1	3	0	-1	-1	1	ŕ	(xy, xz, yz)
$A_{1\mu}$	1	1	1	1	1	-1	-1	-1	-1	-1		
4		1					1					
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1		
$E_u^{A_{2u}}$	1	-1	1	-1 0	-1 0	-1 -2	-1 1	-1 -2	1 0	1 0		
A_{2u} E_u T_{1u}	1 2 3	1 -1 0	1 2 -1	•	-	-1 -2 -3	•	•	1 0 -1	1 0 1	(x, y, z)	
$A_{2u} = E_u = T_{1u} = T_{2u}$			1 2 -1 -1	•	0	-	1	-2		1 0 1 -1	(x, y, z)	

Cr(CO)₆: $A_1 \sim 2100 \text{ cm}^{-1}$, $T_{1u} \sim 2000 \text{ cm}^{-1}$, $E_g \sim 1985 \text{ cm}^{-1}$ Mn(CO)₆⁺: $T_{1u} \sim 2100 \text{ cm}^{-1}$, V(CO)₆⁻: $T_{1u} \sim 1860 \text{ cm}^{-1}$ Question: What combination and overtone bands are possible?

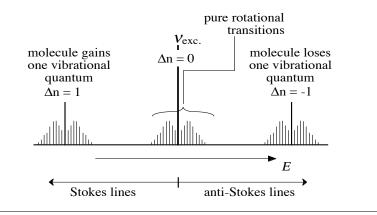
O_h	Ε	8 <i>C</i> ₃	$3C_2 = (= C_4^2)$	$6C_4$	$6C'_2$	i	8 <i>S</i> ₆	$3\sigma_h$	$6S_4$	$6\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
	1	1	1	-1	-1	1	1	1	-1	-1		
$\begin{array}{c} A_{2g} \\ E_g \\ T_{1g} \\ T_{2g} \end{array}$	2	-1	2	0	0	2	-1	2	0	0		$(2z^2 - x^2 - y^2, x^2 - y^2)$
T_{1g}	3	0	-1	1				-1	1	-1	(R_x, R_y, R_z)	
T_{2g}	3	0	-1	-1	1	3	0	-1	-1	1	-	(xy, xz, yz)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		
A_{2u}	1	1	1 2	-1	-1	-1	-1	-1	1	1		
E_u	2 3	-1	2	0	0	-2	1	-2	0	0		
$\begin{array}{c} A_{2u} \\ E_u \\ T_{1u} \\ T_{2u} \end{array}$	3	0 0	-1 -1	1	-1 0 -1 1	-3	0	1	-1	1	(x, y, z)	
1 _{2u}	5	0	-1	-1	1	i -3	0	1	1	-1		



Raman Spectroscopy

• Inelastic scattering of photon (not a resonant absorption spectroscopy)

Example (schematic) - a diatomic molecule in the gas phase



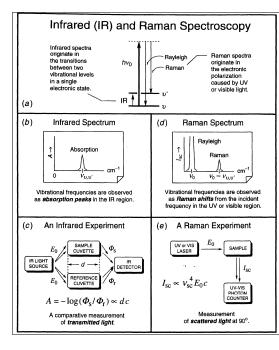


C. V. Raman Nobel Prize in Physics 1930

"for his work on the scattering of light and for the discovery of the effect named after him"

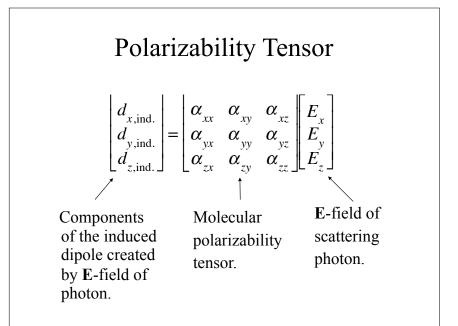
Chandrashekhara Venkata RAMAN 1888-1970

In 1922, Raman published "Molecular Diffraction of Light", first of a series of investigations with his collaborators which ultimately led to his discovery, on the Feb. 28th, 1928, of the radiation effect which bears his name ("A new radiation", *Indian J. Phys.*, **2** (1928) 387.)



IR-Raman Comparison (a) IR = absorption; Raman =

- inelastic scattering
- (b) IR spectrum shows absorption peaks; in Raman vibrational frequencies are observed as shifts from the incident frequency
- (c) in IR an external reference is used for comparison
- (d) in Raman, an intense Rayleigh (elastic scattering) peak is observed with Raman (inelastic) peaks displaced from it - energy difference is absorbed by the molecule
- (e) The exciting radiation must be monochromatic, the signal is proportional to the incident light intensity.



Raman Spectroscopy

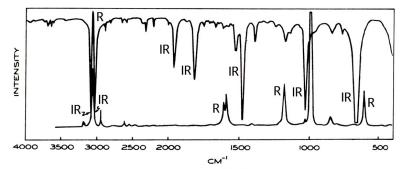
- Became much more important after development of lasers because only about one thousandth of the scattered radiation is Raman, the rest is scattered elastically (with conservation of energy)
- Selection rules scattering amplitudes involve the elements of the polarizability tensor (which must be nonzero):

 $\int \psi_j^{gnd} \alpha_{mn} \psi_j^{exc} d\tau \sim \Gamma_{mn} \otimes \Gamma_j \quad m, n = x, y, \text{ or } z$ i.e., Γ_j must transform as $x^2, y^2, z^2, xy, xz, yz$

Exclusion Rule

- If a molecule is centrosymmetric, then allowed infrared transitions are forbidden in the Raman spectrum and allowed Raman transitions are forbidden in the IR. (Why?)
- If an inversion center is present, then *x*, *y*, and *z* belong to *u* representations and α_{mn} belong to *g* representations.

Exclusion Rule - Benzene



Vibrational spectra of benzene reproduced from S.K. Freeman, Applications of Laser Raman Spectroscopy, John Wiley & Sons, N.Y. 1974.

Examples

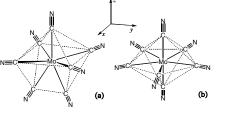
- Describe the number and character of the IR and Raman transitions observed for BrF₅ if the molecule has D_{3h} symmetry. C_{4v} symmetry?
- How can IR and Raman spectra allow one to distinguish between syn- and anti-planar structures of has N₂F₂?
- For BF₃, Raman and IR active modes. B-F stretching modes can be observed in both the IR and Raman spectra which are affected by the presence of two B isotopes (¹⁰B and ¹¹B)?

	D_{3h}	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$			
-	A'_1	1	1			1	1		$x^{2} +$	y^2, z^2
	A'_2	1	1	-1	1	1	-1	R _z		
		2	1	0	2	1	0	(x, y)	$(x^2 -$	y^2, xy
	A_1''			1						
	A_2'' E''	1	1	-1	-1	-1	1	$\begin{vmatrix} z \\ (R_x, R_y) \end{vmatrix}$		
	<i>E</i> "	2	-1	0	-2	1	0	(R_x, R_y)	(x:	z, yz)
4v	Ε	$2C_4$	$C_2($	(C_{4}^{2})	$2\sigma_{v}$	20				
		$\frac{2C_4}{1}$		$\left(\frac{C_4^2}{1}\right)$	$\frac{2\sigma_{v}}{1}$	20 1		Z		$x^2 + y^2, z^2$
1 ₁	1			1				$rac{z}{R_z}$		$x^2 + y^2, z^2$
	1 1	1		1	1 -1	1	1			$\frac{1}{x^2 + y^2, z^2}$ $\frac{1}{x^2 - y^2}$
4 ₁ 4 ₂ 8 ₁	1 1 1 1	1 1 -1 -1		1 1 1 1	1 -1 1 -1	1 - - 1	1	R_{z}		

Example

The seven-coordinate complex $Mo(CN)_7^4$ has been studied both as solid K₄Mo(CN)₇•2H₂O and in aqueous solution. In the C—N stretching region the IR spectrum has bands at 2119, 2115, 2090, 2080, 2074, and 2059 cm⁻¹ for the solid and at 2080 and 2040 cm⁻¹ for solutions. How many Raman and infrared bands would you expect for (a) the monocapped trigonal

prism and (b) the pentagonal bipyramid and how many coincidences (bands present in both the IR and Raman spectra) should there be for each geometry? How do you interpret these data?



Fundamentals and Overtones and Combination bands for XY₄ *molecules*

- On p. 194 of Carter, overtones of CH₄ are treated incorrectly. Cotton leaves out their treatment entirely. Harris and Bertolucci provide a decent discussion (p. 164-7 and Appendix C).
- Let's work out the normal modes, look at the fundamentals - then discuss how the overtone and combination bands are handled differently

Overtones - Degenerate Modes; Symmetric Direct Products

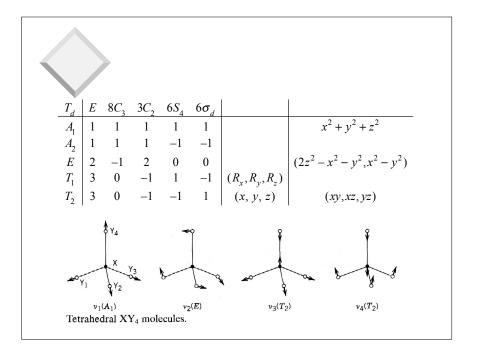
• With two quanta in a degenerate mode, the symmetries of Ψ_{vib} are given by*:

$$\chi^{+}(R) = \frac{1}{2} (\chi^{2}(R) + \chi(R^{2}))$$

• With three quanta the formula is

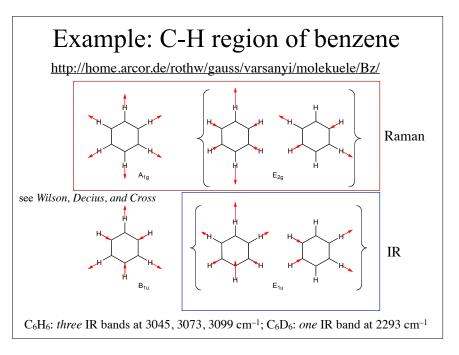
$$\chi^{+}(R) = \frac{1}{6} \left(\chi^{3}(R) + 3\chi(R)\chi(R^{2}) + 2\chi(R^{3}) \right)$$

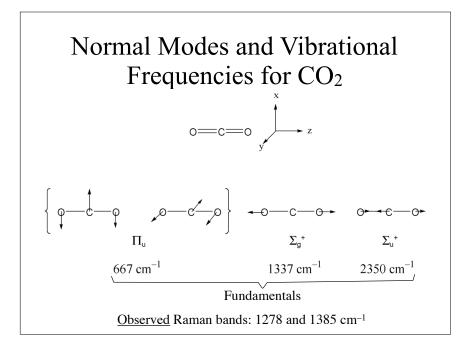
(*See Wilson, Decius, & Cross, "Molecular Vibrations...", Section 7.3; this is the source material for Harris & Bertolucci's Appendix C. Also see the *Antisymmetry* handout for proof for the two and three quanta cases.)

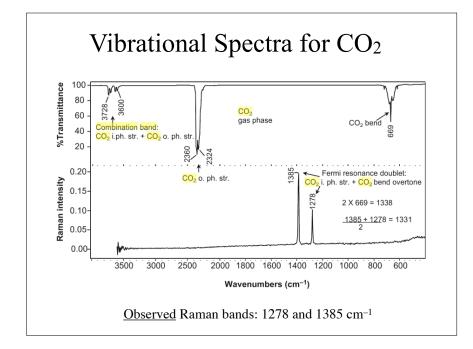


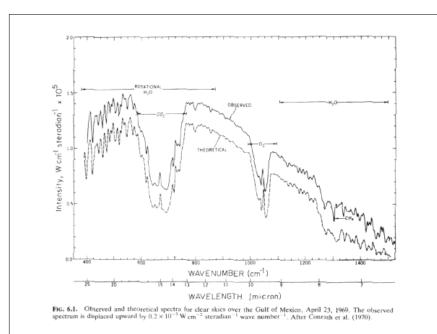
Fermi Resonance

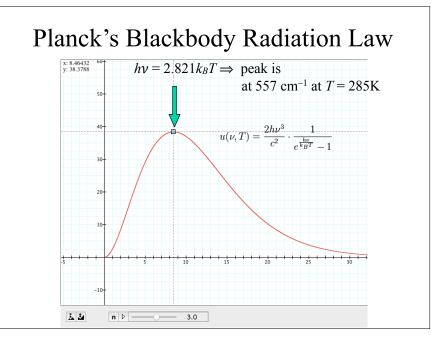
- Just as "configuration interaction" occurs in electronic spectra, mixing between fundamentals and combination or overtone bands can occur in vibrational spectra.
- 'Weak' combination/overtone bands can 'borrow' intensity from fundamentals if the sum of the frequencies in the combination/ overtone band is close to the frequency of the fundamental.
- CO₂ example from Cotton: fundamental frequencies are 668, 1337 (Raman), 2349 cm⁻¹ (what are they?). 200 mixes with 010.











n.

			A	ctive
Group	Bands (cm ⁻¹)	Assignment	IR	Ramar
CO ₂	1385, 1278	$\frac{\text{CO}_2}{\text{CO}_2}$ i. ph. str + 2x $\frac{\text{CO}_2}{\text{CO}_2}$ bend	No	Yes
R-C(=0)-H	~2820, ~2710	CH str. + 2x CH i.ph. bend	Yes	Yes
$N=C=O^{-}$	~1300, ~1200	NCO i.ph. str. + 2x NCO bend	Yes	Yes
R-COOH	~3000-2650	OH str. $+ 2x$ OH i. pl bend	Yes	Yes
	~3000-2550	OH str. + 2x C-O str.	Yes	Yes
CH ₂	~2920-2890	$CH_2 str. + 2 \times CH_2 bend$	Yes	Yes
Aryl-C(=O)-Cl	~1780, ~1730	C=O str. + 2x 880 aryl-C str.	Yes	Yes
$N \equiv C - N = C(NH_2)_2$	2200, 2170	$C \equiv N \text{ str.} + (1250 + 925 \text{ cm}^{-1})$ Combination band	Yes	-
$R-C\equiv C-R$	2300, 2235	$C \equiv C \operatorname{str} + C - C \equiv C \operatorname{bend}$	No	Yes
C=O	1810, 1780	C=O str. + 2x ~900 ring bend	Yes	-
Aryl-NH ₂	3355, 3205	NH_2 i. ph. str. + 2x NH_2 bend	Yes	Yes
NH ₄ Cl	2825	NH_4 i. ph. str. $+ 2x NH_4$ bend	Yes	Yes
-C(=O)-NH-C	3300 (s), 3100(m)	MH str. + 2x CNH strbend	Yes	Yes

$D_{\infty h}$	Ε	$2C^{\Phi}_{\infty}$		$\infty \sigma_{v}$	i	$2S^{\Phi}_{\infty}$		∞C_2		$ \begin{array}{c} x^{2}, y^{2}, z^{2} \\ (xz, yz) \\ (x^{2} - y^{2}, xy) \end{array} $
Σ_g^+	1	1		1	1	1		1		x^2, y^2, z^2
Σ_{g}^{-}	1	1		-1	1	1		-1		
П	2	$2\cos\Phi$		0	2	$-2\cos\Phi$		0	R_z	(xz, yz)
Δ_g	2	$2\cos 2\Phi$		0	2	$2\cos 2\Phi$		0	(R_x, R_y)	$(x^2 - y^2, xy)$
							<u></u>			
Σ_u^+	1	1		-1	-1	-1 1 $2\cos\Phi$ $-2\cos 2\Phi$	•••	1	z	
Σ_u^-	1	-1		-1	-1	1		1		
П	2	$2\cos\Phi$		0	-2	$2\cos\Phi$		0	(x,y)	
Δ_u	2	$2\cos 2\Phi$		0	-2	$-2\cos 2\Phi$	•••	0		
erefore,		,	- ($\cos 2\Phi = (1 + 2\cos 2\Phi)$				$1+2\cos 2\Phi$